CHAPTER 2. MULTILEVEL PATH MODELS

In the previous section, we offered a quick overview of both structural equation models and multilevel models, as well as how both the modeling and the notation can be merged into a single framework. Up to this point, the most common method for the analysis of hierarchical data structures was limited to regression-like modeling situations. This chapter generalizes a more complex structure of relationships to a multilevel framework. Path models are arguably the most simple structural equation models, incorporating only observed variables but going beyond the situation with only one endogenous variable and multiple exogenous ones. Here, we will generalize such models to the multilevel framework.

Simplicity often goes hand in hand with flexibility, and path models have been used for a wide range of questions. They have proved to be especially powerful and insightful when combined with a strong argument for what the proper causal ordering should be among variables in a model. A case in point is Blau and Duncan's (1967) celebrated account of intergenerational mobility within the occupational hierarchy of the United States. The authors make a strong case for parental education and occupation as pure exogenous factors, which come to influence offspring's education and their first job (endogenous factors). Through these transmission mechanisms but also directly, they come to shape the second generations' current position in the occupational hierarchy (see Blau & Duncan, 1967, fig. 5.1). Through the use of path modeling, the authors are able to determine the relative contribution of parental factors and of personal effort to the process of occupational stratification, as well as the pathways through which parental factors operate.

A further feature of this modeling strategy is its ability to capture reciprocal effects via nonrecursive specifications. An early example of this is the Duncan, Haller, and Portes (1971) model of the influence of peers on professional aspirations. Here, both personal intelligence and family socioeconomic status (SES) shape a person's professional aspirations. The corresponding pair of factors naturally shape a peer friend's aspirations. However, the authors also introduce the potential for a friend's family SES to influence aspirations through the influence of role models, as well as for a friendship dyad's aspirations to shape each other. Parsimonious and elegant, this and the following specifications used by the authors allow them to disentangle how much of a person's professional aspirations are due to personal factors, role models' influence, or peer examples. A final example reveals an oft-forgotten strength of path modeling: the ability to estimate off-diagonal cells in a variance-covariance matrix under the assumption of a properly specified model and then use them further on in the estimation of the model. This is done by Duncan (1968) through the use of multiple sources of data with the goal of estimating cells in the variance-covariance matrix. For the cells where no data source could be used to obtain a measurement due to missing data, an application of path analysis rules produces these covariances based on information already available in the matrix and the model specification (Duncan, 1968, p. 7). As a final step, the model is then estimated using the variance-covariance matrix. These features of path analysis are very portable and powerful in situations where reciprocal effects or associations between predictor variables are suspected to operate.[1]

Before getting to the substance of the chapter, we urge the reader to revisit the notation conventions presented in Figure 1.1 for SEM models and in Figure 1.4 for MLM models. These ways of graphically describing an MSEM model will be used consistently from now on in most of the specifications we discuss. We also note here that while we sometimes use the terms *predictor* and *predict* to refer to exogenous covariates of a variable and their effect, we do not imply a causal ordering through the use of this language.

We start with an example from Wave 4 of the *World Values Surveys* (WVS), a cross-cultural survey incorporating a reasonably large number of countries for multilevel analysis. Our interest is in citizens' self-expression values (available in the WVS data), which occupy a central mediating role in a theorized chain of associations that starts with economic development and ultimately produces democratization (see Inglehart & Welzel, 2009). Such values signal that the individual assigns high importance to participation in decision making, to expression of one's individuality as opposed to conformism, to environmental responsibility, and to tolerance of alternative lifestyles. As a theoretical curiosity, but also as a practical question, we are interested in who are the individuals most likely to exhibit high levels of such self-expression values. Given their heightened predisposition to press authoritarian regimes for increasing political openness, identifying individual-level

[1] The reader can find more examples like these in Wolfle (2003).

Table 2.1 List of Variables From *World Values Surveys* Example

Code	Item	Response Scale
Individual level		
SEV	Emphasis on the importance of civic activism, subjective well-being, tolerance and trust, personal autonomy, and choice (Inglehart & Baker, 2000)	Scale approx. ranging in the sample from −1 to 3.16
AGE	Respondent's age	Numeric, ranging from 15 to 101
INC	Respondent's household income	Ordinal scale, ranging from 1 to 10 (deciles)
EDU	Respondent's highest educational level	Ordinal scale, ranging from 1 ("inadequately completed elementary") to 8 ("university with degree")
Country level		
GDP	Country's gross domestic product/capita, adjusted by purchasing power parity, in current international dollars	Numeric, ranging in the sample from approx. 1,000 to 75,000

Note: Original variable names from the WVS data set are `survself` (*SEV*), `X003` (*AGE*), `X047` (*INC*), and `X025` (*EDU*). Original variable name from QoG data set, January 2016 version, is `wdi_gdppcpppcur` (*GDP*).

factors connected to such values can help us better explain the appearance of pressures for change in a country. At a deeper level of analysis, also identifying the systemic characteristics associated with a greater preponderance of such values could help advocacy organizations better target their democracy promotion efforts.

The multilevel data structure at hand is one where individuals are grouped within countries. The variables at Level 1 are, first, a constructed scale of self-expression values, available in the original data set; here, higher scores denote an individual's greater emphasis on self-expression. We also have in the data an individual's income, ranging from 1 to 10 (in income deciles), as well as age (measured in decades and rescaled, so that 0 denotes 1.8 decades) and the highest educational level attained (a variable with eight ordered categories). At the country level, gross domestic product (GDP) per capita, adjusted by purchasing power parity (PPP), expressed in constant international dollars, is obtained from the *World Development Indicators*. Following conventions in the literature, the natural logarithm of GDP was used to eliminate model convergence issues, achieve normality of the variable and potentially also the residuals, and ensure that the relationships are closer to linear. In the interest of simplicity, all models were estimated on a sample of 42,619 respondents from 55 countries. This was ensured by performing listwise deletion for missing information prior to estimation.

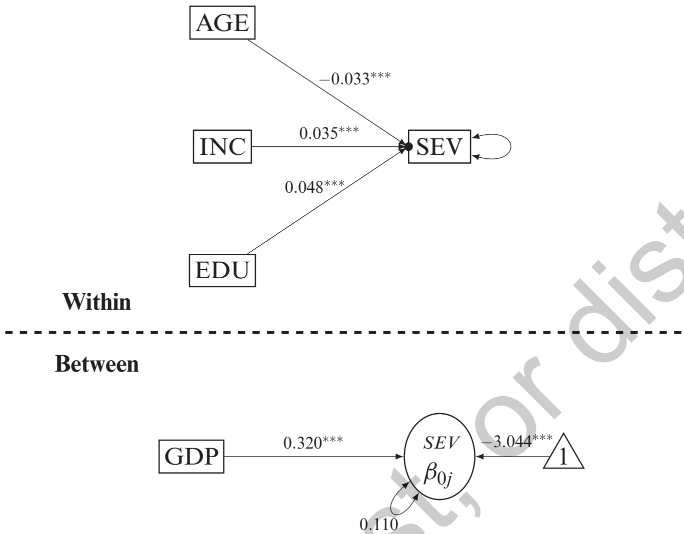
Our example is admittedly pared down, with only four exogenous covariates for self-expression values. The primary reason for this is the need to keep model specifications and figures at a manageable size and complexity. At the same time, though, our specification captures the core features of most real-life MSEMs. As the reader will see in the following sections, even with such a reduced model, we are able to offer tentative answers to what are the individual-level and country-level factors associated with high self-expression values. It is important to note that the example could be expanded with the addition of further constructs of interest into the model, but the amount of results to be interpreted would grow along. In summary, we find the specification to be a suitable teaching device but falls far short of a causally identified and properly specified model for self-expression values at the individual level.

Multilevel Regression Example

When confronted with a hierarchical data structure and with the previously mentioned set of theoretical questions, the first instinct of the applied data analyst would likely be to use a standard multilevel model. With this data configuration, the analyst can be confident that the standard errors produced by the model are accurate and that any effect of contextual exogenous covariates on the individual-level dependent variable (DV) would be accurately estimated. A very simple model, for demonstration purposes, might be to regress self-expression values on income, education, and age at the individual level and GDP per capita at the country level. This is precisely the type of model used by Welzel and Inglehart (2010) in their investigation of what drives self-expression values.

A presentation of the model in diagram form, along with the estimated parameters, can be seen in Figure 2.1. The results are plausible, albeit based on an underspecified model. All three exogenous covariates are statistically significant, with effects in the expected direction:

Figure 2.1 Standard Multilevel Regression Model Specification (unstandardized results reported)



Individuals with a higher level of education exhibit, on average, a greater extent of self-expression values, as do individuals with higher incomes. In a similar way, older individuals display lower levels of such values, although we are unable to say based on this specification whether we are dealing with an age or a cohort effect. More important, although we only use one wave of the WVS, the effects we find are in the range of what Welzel and Inglehart (2010) find, even though their model is slightly more complex and is estimated on three WVS waves. In addition, at the Level 2, GDP per capita *positively* affects the extent to which an individual manifests self-expression values: Wealthier countries also display higher average levels of self-expression values in the population. Further work certainly awaits the applied modeler: testing alternative specifications, arriving at a best-fitting model, inspecting residuals, and so on. Should the final model pass all "quality checks," however, the investigation of the modeler concludes with the final interpretation of coefficients.

It is important to point out that the alternative specifications tried by our researcher all presume direct associations between each exogenous covariate and the outcome, and all estimate direct effects of these variables on the outcome. In many situations, though, this approach clearly ignores linkages that can exist among exogenous variables themselves.

In standard analyses of turnout at the individual level, education is an explanatory factor for turnout and for political efficacy, itself a determinant of turnout. It is also plausible to conceive of social class as directly explaining party choice, due to exposure to party mobilization efforts or pressure from social networks, as well as predicting issue position, which itself comes to guide party choice. Finally, in our simplified example, age is not solely an explanatory factor for self-expression values but also for the level of education attained.

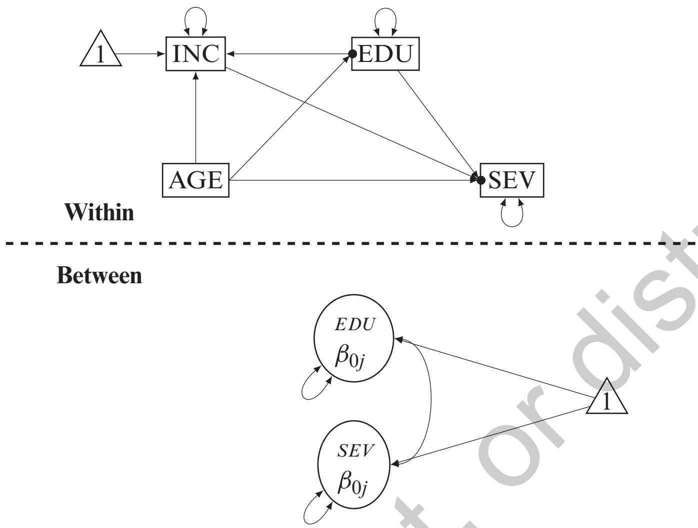
In the case of such direct and indirect statistical associations, the standard modeling approach for the past four decades has been to employ a structural equation model. In such a specification, income, education, and age could not only explain self-expression values but also be connected to each other (e.g., age to education). This effectively turns education from a purely exogenous predictor to an endogenous one. While the theory and estimation routines behind the standard SEM approach are solidly established, these would be of only limited use in our case. The presence of data on multiple level of analysis, with individuals clustered in countries, means that standard errors will be inefficient, and by implication, significance tests produced by the standard SEM toolbox would be imprecise. Using the example of GDP per capita, our analysis would treat all 42,619 members of the sample as contributing unique pieces of information to the final estimated quantity and its uncertainty. However, this is clearly wrong, as for *GDP* we only have 55 measurements, one for each country in our sample. While not as serious, the same problem plagues the estimates for other exogenous and endogenous variables in our model.

The analyst is then faced with a dilemma: either obtain accurate estimates of effect and uncertainty through an MLM model, at the cost of ignoring the larger structure of associations in the model, or model this structure properly through the use of a SEM, at the cost of estimates that ignore the data clustering. In the following sections, we present a few model specifications that allow the researcher to overcome the dilemma posed here and add additional modeling flexibility through the inclusion of variables (both causes and consequences) at the second level of analysis.

Random Intercepts Model

The model specification upon which we base our initial discussion is presented in Figure 2.2, depicting a standard setup for a multilevel path model. A respondent's position on the self-expression values scale is regressed on income, age, and education, while age is a covariate of

Figure 2.2 Standard Path Model



education, and education and age are explanatory factors for a person's income. In this model, therefore, income, education and self-expression values are endogenous variables while age is exogenous. The structure of relationships is presented in notation form in Equation 2.1. In a sense, this is a mediation model (Iacobucci, 2008) where the impact of education and age on self-expression values is mediated by income and education, respectively. In this model, we also control for the impact of age on income, which only has an indirect effect on self-expression.

$$\begin{cases} SEV_{ij} = \overset{SEV}{\beta_{0j}} + \overset{SEV}{\beta_{1j}} INC_{ij} + \overset{SEV}{\beta_{2j}} EDU_{ij} + \overset{SEV}{\beta_{3j}} AGE_{ij} + \overset{SEV}{\varepsilon_{ij}} \\ INC_{ij} = \overset{INC}{\beta_{0j}} + \overset{INC}{\beta_{1j}} EDU_{ij} + \overset{INC}{\beta_{2j}} AGE_{ij} + \overset{INC}{\varepsilon_{ij}} \\ EDU_{ij} = \overset{EDU}{\beta_{0j}} + \overset{EDU}{\beta_{1j}} AGE_{ij} + \overset{EDU}{\varepsilon_{ij}} \end{cases} \tag{2.1}$$

At this point, the model at hand is nothing more than a structural equation model. But we know that individuals in the data set are from their respective countries. To account for the potential bias that emerges from hierarchical data, if ignored, we allow the intercepts in this model to vary across countries. We believe that although the impact of age on education and that of education on self-expression values is roughly the same for each country, the baseline levels of education and self-expression values are different between countries. We hope that most readers would consider this statement plausible, at least with respect to education. For this reason, these intercepts are allowed to vary across countries. It could also be argued that the variances of income could be interesting, but given the standardization of the variable into deciles, the interpretation of the effect (especially on the country level) would be quite difficult. Hence, for the purposes of this exercise, we are treating this variable as a control and not as one with substantive interest, and we are omitting all estimated random effects.

Here we allow the variance of the above mentioned intercepts at the between-country level. By doing so, in MSEM estimation, we split the total covariance matrix into two: one within and one between clusters. They are additive (meaning, the total covariance is the sum of within and between covariances) and uncorrelated (B. O. Muthén, 1994). We also add a predictor on the country level to explain variation in both of the varying intercepts: GDP per capita. The six slopes at the Level 1 (one for income, two for education, and three for age) are not allowed to vary between countries. These relationships are presented in Equation 2.2, where we follow the convention introduced by Snijders and Bosker (1999) of denoting fixed intercepts and slopes with a Level 2 γ rather than a Level 1 β.

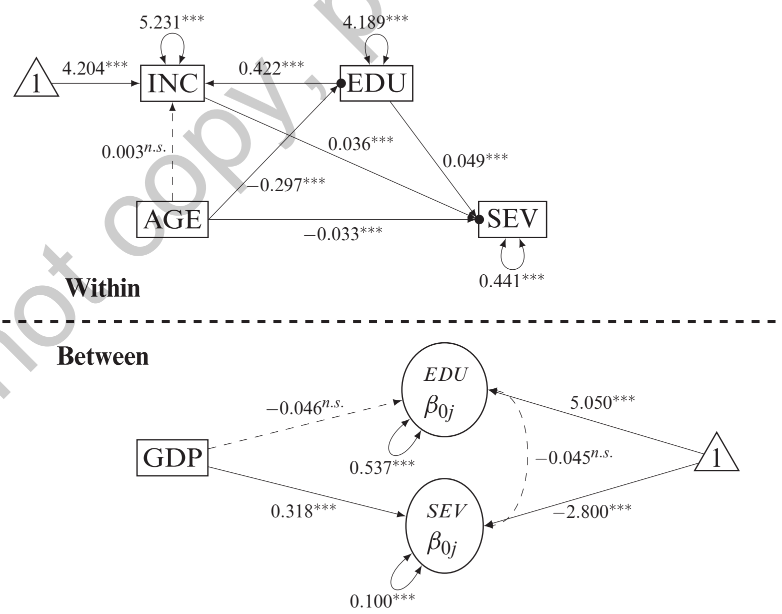
$$\begin{cases} \overset{SEV}{\beta_{0j}} = \overset{SEV}{\gamma_{00}} + \overset{SEV}{\gamma_{01}} GDP_j + \overset{SEV}{\upsilon_{0j}} \\ \overset{SEV}{\beta_{1j}} = \overset{SEV}{\gamma_{10}} \\ \overset{SEV}{\beta_{2j}} = \overset{SEV}{\gamma_{20}} \\ \overset{SEV}{\beta_{3j}} = \overset{SEV}{\gamma_{30}} \\ \overset{INC}{\beta_{0j}} = \overset{INC}{\gamma_{00}} \\ \overset{INC}{\beta_{1j}} = \overset{INC}{\gamma_{10}} \\ \overset{INC}{\beta_{2j}} = \overset{INC}{\gamma_{20}} \\ \overset{EDU}{\beta_{0j}} = \overset{EDU}{\gamma_{00}} + \overset{EDU}{\gamma_{01}} GDP_j + \overset{EDU}{\upsilon_{0j}} \\ \overset{EDU}{\beta_{1j}} = \overset{EDU}{\gamma_{10}} \end{cases} \tag{2.2}$$

The extended form of the model, for each Level 1 equation, then becomes the specification shown in Equation 2.3.

$$
\begin{cases}
SEV_{ij} = \overset{SEV}{\gamma}_{00} + \overset{SEV}{\gamma}_{10} INC_{ij} + \overset{SEV}{\gamma}_{20} EDU_{ij} + \overset{SEV}{\gamma}_{30} AGE_{ij} + \\
\qquad + \overset{SEV}{\gamma}_{01} GDP_j + \overset{SEV}{\upsilon}_{0j} + \overset{SEV}{\varepsilon}_{ij} \\
INC_{ij} = \overset{INC}{\gamma}_{00} + \overset{INC}{\gamma}_{10} EDU_{ij} + \overset{INC}{\gamma}_{20} AGE_{ij} + \overset{INC}{\varepsilon}_{ij} \\
EDU_{ij} = \overset{EDU}{\gamma}_{00} + \overset{EDU}{\gamma}_{10} AGE_{ij} + \overset{EDU}{\gamma}_{01} GDP_j + \overset{EDU}{\upsilon}_{0j} + \overset{EDU}{\varepsilon}_{ij}
\end{cases}
\tag{2.3}
$$

For convenience, the model is also presented in graphical form in Figure 2.3, with estimates from the model included. Much like the simple multilevel model, we see that age exerts a direct effect on self-expression values ($\overset{SEV}{\gamma}_{30} = -0.033^{***}$). But it is also clear that some originally independent variables of *SEV* turn into endogenous variables themselves; for instance, age has a significant impact on education ($\overset{EDU}{\gamma}_{10} = -0.297^{***}$).

Figure 2.3 Multilevel Path Model With Random Intercepts (unstandardized results reported)



This potentially accounts for some indirect effects the multilevel model completely misses, since it only estimates direct relationships between each covariate and the outcome. Here, this is clearly not the case. The negative effect of age on education is likely explained by the fact that educational opportunities have only recently expanded in a substantial number of countries in our sample. This means that younger people in the population are, on average, more educated than those who are older, leading to the negative estimate we observe.

Moving on to the between level, at the bottom part of Figure 2.3, in this model specification, GDP per capita has a positive impact on the intercept of self-expression values (or, we can also say, directly affects self-expression values) but does not affect educational attainment. It would appear, then, that wealthier countries have a higher level of self-expression values, even after we control for individual-level factors, including income. However, this effect is not due to higher average levels of education in these countries, as richer and poorer countries have roughly similar levels of educational achievement. One explanation might reside in the type of education that richer and poorer countries tend to emphasize. We speculate that wealthier countries emphasize a liberal arts education to a greater degree than do poorer ones. This promotes some of the attitudes that constitute the self-expression cluster, such as self-expression and importance allocated to participation. Lower GDP countries, on the other hand, emphasize this type of education less. In turn, their curriculum is geared more toward memorization and exact sciences. To sum up, we suspect that it is the content of education that is different in countries at different GDP levels, rather than the absolute number of years of education.

For full disclosure, we need to note that the figure has one additional estimated parameter that is not represented in the equation in the interest of parsimony and simplicity. This is the covariance between the intercepts of *EDU* and *SEV* that are allowed to vary across Level 2 units, depicted in the between part of Figure 2.3 by the curved arrow connecting the circles (latent indicators). What is important to remember is that covariances between these Level 2 latent variables that emerge from the variance components of Level 1 intercepts (or slopes, as seen in the next section) can be allowed to covary or they can be fixed to 0, forcing no relationship. We believe the decisions should rest on theoretical expectations, although another school of thought suggests that seeking the appropriate balance between parsimony and model fit should drive these decisions, if need be, atheoretically.

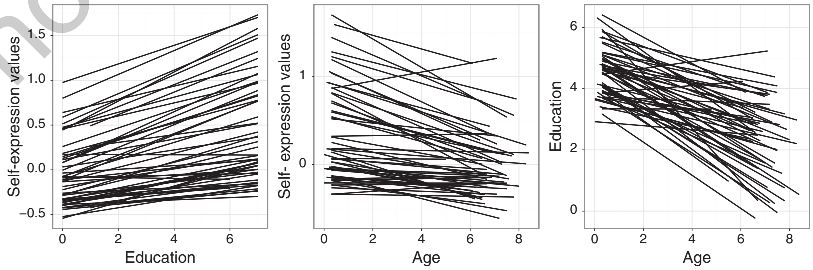
We are confident in the added power of MSEM specifications in the case of hierarchical data structures, compared to MLMs or SEMs, irrespective of the actual conclusions drawn from the data. It is true that the individual-level effects of age, education, and income are roughly similar to those displayed in the previous model. At the same time, such a model specification allows us to offer a richer description of the effect pathways that operate in reality. We now see that age has both a direct effect on self-expression values, as well as an indirect effect, through education. The same can be said of the effect of education on self-expression values.

Random Slopes Model

The model specification presented in Figure 2.3 captures a snapshot of the relationships between self-expression values and age, income, and education, along with GDP per capita at the country level. The key insight from the model is that GDP per capita is associated with the level of self-expression values in a country, suggesting that bottom-up pressures for democratic change could be more likely in wealthier countries (Epstein, Bates, Goldstone, Kristensen, & O'Halloran, 2006; but see Przeworski, Alvarez, Cheibub, & Limongi, 2000 for findings that go against this assertion). Furthermore, within countries, it is younger and more educated citizens who are more likely to harbor these values.

Nevertheless, we contend that there is yet more to discover about the dynamics comprising the data-generating process. Breaking down the relationships between age, education, and self-expression values, we find heterogeneity in the effects of age and education (see Figure 2.4). While

Figure 2.4 Bivariate Relationships Between Education, Self-Expression Values, and Age (individual country fit lines)

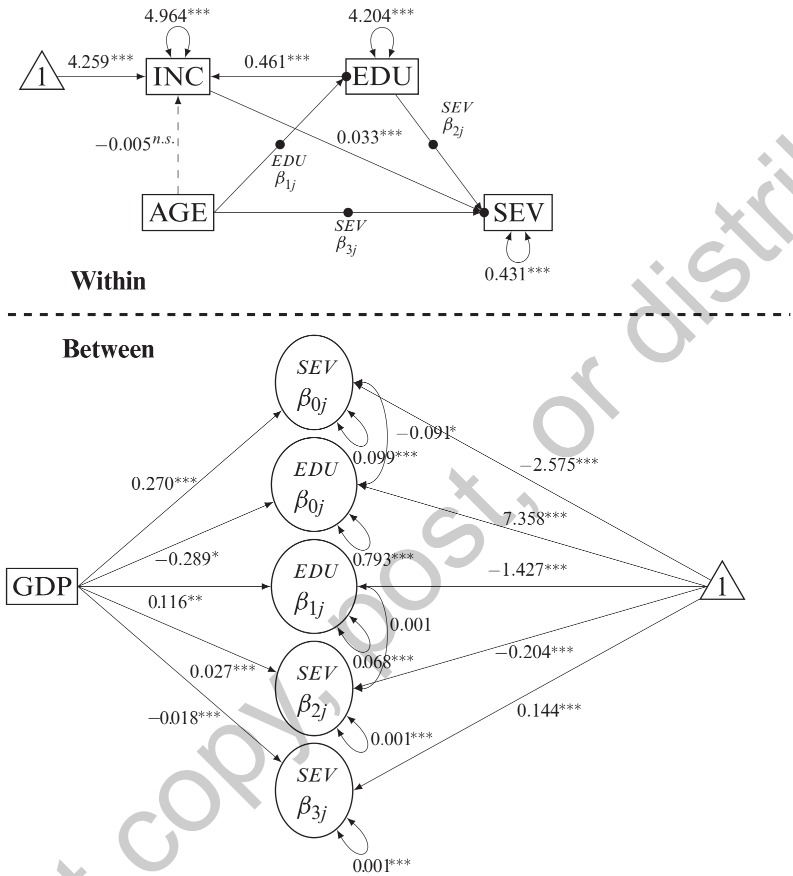


the effect of education on self-expression values is mostly positive, it clearly has varying strength, with a maximum of 0.162 in the Netherlands and a minimum of –0.005 in Uganda (India is the only other country where this effect is negative). In a similar manner, for education, the effect is predominantly negative, with a minimum of –0.162 in Denmark. At the same time, there are clearly situations where this effect is positive, such as Algeria, Hungary, Moldova, or the United States (six countries in total). Finally, the effect of age on education is predominantly negative: Older individuals are, on average, less educated, presumably due to the more restricted educational opportunities available to them when they were transitioning to adulthood. The strongest negative effect is found in Algeria (–0.876). There are, however, cases where this effect is positive (United States), as well as contexts with virtually no effect (the Czech Republic, Tanzania, or Uganda). As a result of this, in our final specification, we have also allowed these relationships to (randomly) vary across countries and have added GDP per capita as an explanatory factor for this variance. GDP per capita is a potential moderator for the slope of education on self-expression values, for the slope of age on self-expression values, and for the slope of age on education. In addition, as in the previous specification, the intercepts of education and self-expression values have been allowed to vary across countries and are predicted by GDP per capita as well. This is actually necessary; when a slope is allowed to vary across the Level 2 units, it is important to allow the intercept to vary as well. For this reason, a random slopes model is also always a random intercept model despite the fact that we just call it a random slopes model for short.

Figure 2.5 presents a graphical depiction of this model, along with the results of the estimation procedure. In Equation 2.4, we only present the extended form of the specification.

$$
\begin{aligned}
SEV_{ij} &= \overset{SEV}{\gamma}_{00} + \overset{SEV}{\gamma}_{10} INC_{ij} + \overset{SEV}{\gamma}_{20} EDU_{ij} + \overset{SEV}{\gamma}_{30} AGE_{ij} + \\
&+ \overset{SEV}{\gamma}_{21} GDP_j EDU_{ij} + \overset{SEV}{\gamma}_{31} GDP_j AGE_{ij} + \overset{SEV}{\gamma}_{01} GDP_j + \\
&+ \overset{SEV}{\upsilon}_{2j} EDU_{ij} + \overset{SEV}{\upsilon}_{3j} AGE_{ij} + \overset{SEV}{\upsilon}_{0j} + \overset{SEV}{\varepsilon}_{ij} \\
INC_{ij} &= \overset{INC}{\gamma}_{00} + \overset{INC}{\gamma}_{10} EDU_{ij} + \overset{INC}{\gamma}_{20} AGE_{ij} + \overset{INC}{\varepsilon}_{ij} \\
EDU_{ij} &= \overset{EDU}{\gamma}_{00} + \overset{EDU}{\gamma}_{10} AGE_{ij} + \overset{EDU}{\gamma}_{01} GDP_j + \overset{EDU}{\gamma}_{11} GDP_j AGE_{ij} + \\
&+ \overset{EDU}{\upsilon}_{1j} AGE_{ij} + \overset{EDU}{\upsilon}_{0j} + \overset{EDU}{\varepsilon}_{ij}
\end{aligned}
\tag{2.4}
$$

Figure 2.5 Multilevel Path Model With Random Intercepts and Slopes (unstandardized coefficients)



The first change, as compared to the random intercepts model, is that the impact of *GDP* on education becomes significant. Allowing the slopes to vary also affected the variance of the intercepts of education, resulting in an improved ability of *GDP* to explain this variance ($\gamma_{01}^{EDU} = -0.289^{*}$). GDP per capita also has a statistically significant moderation effect on the impact of education on self-expression values ($\gamma_{21}^{SEV} = 0.027^{***}$), as well as on the slope of age when it is regressed on education ($\gamma_{01}^{EDU} = 0.117^{**}$). Finally, *GDP* also significantly influences the relationship between age and self-expression values ($\gamma_{31}^{SEV} = -0.018^{***}$).

For example, while in extremely poor countries, the effect of education on self-expression values is negative, the effect turns positive in wealthier countries. At the same time, though, in poorer countries, the effect of age on education is negative and relatively small. In wealthier countries, the effect of age on education is larger and even further in a negative direction. In essence, in wealthier countries, a reasonably large amount of education is the norm today, whereas this was not the case a mere five decades ago. For this reason, older people often have substantially less education than do younger generations. The same phenomenon is going on in poorer countries, but the variance in education for current generations is still quite high. For example, in societies where a high percentage of the population works in agriculture, people often stay away from schools to help run the family farm. While this could happen in wealthier countries as well, it is more common to send the person off for an agricultural degree first. In addition, fewer people stay in the more traditional sectors where continuing the family business requires no formal education. These are plausible reasons why the effect is weaker in poorer countries and stronger in wealthier ones. The first finding presented here could have been revealed by a standard MLM analysis; the second one could have only been formally tested in an MSEM model.

Further, note that the only Level 2 covariance that is included is the one that was estimated in the random intercepts model. If theoretically relevant, one could test additional covariance components, such as the relationships between the slopes or the relationships between the intercepts and the slopes. After careful theoretical development, we could potentially hypothesize how the intercept of one of the endogenous variables (such as income) is related to the relationship between education and self-expression values when considered on the country level. While these deep theoretical considerations have been sparse even in the multilevel regression modeling literature, that did not stop the method from becoming immensely popular among social scientists. As is also the case with multilevel models, with MSEM, our empirical models allow for greater flexibility than is normally encountered in our theoretical frameworks. However, to test such a relationship, we should not forget that our sample size at the country level is still 55, and a convenience sample at that, and that we are working with variance estimates that are already unstable due to the limited sample size. These estimates are extremely underpowered and hence quite unreliable even when statistically significant results are found. Unfortunately, larger samples are inconceivable for research using cross-country surveys in the absence

Table 2.2 Model Fit Statistics

	Deviance ($-2LL$)	AIC	BIC	Parameter
Random intercepts	460,015.6	460,049.6	460,196.8	17
Random slopes	457,398.7	457,444.7	457,643.9	23

Note: Estimates from the random intercepts model are displayed in Figure 2.3, while those from the random intercepts and random slopes model are found in Figure 2.5.

of enough countries in the world but are very plausible when studying voters in precincts, students in classrooms, or patients of doctors, for example.

Comparison of Random Intercepts and Random Slopes Models

Comparing the random intercepts model with the random slopes model in Table 2.2, we get the sense that the latter represents an improvement in fit compared to the former. The deviance for the random intercepts model is 460,015.6, while for the random slopes model, it is 457,398.7. The difference between the two is 2,616.9, which is highly statistically significant when considering that the critical value for a χ^2 distribution with $23 - 17 = 6$ degrees of freedom is 12.59. This suggests that the random intercepts model fits the data significantly worse than the random slopes model. Both the AIC and the BIC reinforce this conclusion as they are considerably smaller for the latter.

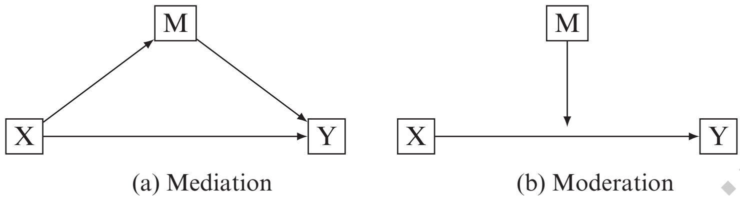
Mediation and Moderation

One of the powers of MSEM over a simple multilevel regression model is our ability to test the "structure" of relationships that go beyond one dependent variable associated with multiple independent variables. Through this newly gained flexibility, we can test relationships of mediation inside our structure of associations, which opens up the potential for gauging direct and indirect effects of covariates of interest.

Mediation

A relationship of *mediation* exists in instances where an association between an exogenous covariate and an outcome can be shown to be transmitted through a third variable, the mediator. While moderation

Figure 2.6 Basic Framework for Statistical Mediation and Moderation



explains variations in the strength of the association between a covariate and an outcome, mediation will give a precise account of how this association is transmitted, under the form of a specific *transmission mechanism* (Baron & Kenny, 1986). Figure 2.6a depicts this instance, with M acting as a mediator in the relationship between X and Y. While providing a richer description of how social phenomena unfold, mediation also relies on a set of more stringent assumptions. The pathways depicted between X, M, and Y require that the researcher defend a strict temporal ordering, with X causally prior to M and M causally prior to Y.

When discussing the simple mediation framework (Baron & Kenny, 1986) in the context of multilevel models, the wide variety of configurations that can be produced with only three variables, X, M, and Y, cause difficulties for the standard MLM setup. Even a standard configuration, such as a Level 1 mediator for the relationship between a Level 2 exogenous covariate and a Level 1 outcome, leads to problems of confounding of within-group effects (from M to Y) and between-group effects (from X to M), as pointed out by Zhang, Zyphur, and Preacher (2009). Yet possibilities abound beyond this $2 \rightarrow 1 \rightarrow 1$ situation, if we use the notation convention introduced by Krull and MacKinnon (2001), where the numbers denote the level at which a certain variable in the mediation chain is. We could have instances where a Level 1 M mediates the association between a Level 1 X and a Level 2 Y ($1 \rightarrow 1 \rightarrow 2$). These *micro-macro* effects (Snijders & Bosker, 1999), along with a variety of $2 \rightarrow 1 \rightarrow 2$, $1 \rightarrow 2 \rightarrow 2$, or even $1 \rightarrow 2 \rightarrow 1$ configurations, cannot be tested at all in a standard MLM setup, due to its inability to accommodate an outcome variable at the Level 2 (Preacher, Zyphur, & Zhang, 2010, p. 211). This is where the MSEM framework proves particularly useful.

In the example multilevel structural equation models presented, we have multiple relationships that could have both direct and indirect effects on self-expression values. One clear example is the relationship between age and self-expression values. In the multilevel regression model of Figure 2.1, we see that the impact of age on self-expression values is -0.033^{***}. The estimate based on the random intercept model in Figure 2.3 is actually identical. But Figure 2.3 highlights that the relationship between age and self-expression values can go through education, given that age and education are related. So, in addition to the direct effect of age, obtained after partialling out the influence of education and income, there also exists an indirect effect of age, transmitted through education.[2] In this specific instance, then, we are dealing with a $1 \rightarrow 1 \rightarrow 1$ mediation setup. In a single-level SEM, we would use tracing rules to find the indirect effect of age on self-expression values through education. It would simply be the product of the coefficient from age to education and education to self-expression values (the *product-of-coefficients* method).

However, in a multilevel setting, the two random slopes coefficients are assumed to be random variables with a bivariate normal distribution, which means that applying this simple rule would lead to biased estimates (Kenny, Korchmaros, & Bolger, 2003). In practice, the expected value of an indirect effect is the multiplication of the two coefficients plus their covariance (Goodman, 1960; Kenny et al., 2003). Therefore, to find the indirect effect of age on self-expression values, we look at the product of the mean effect from age to education and education to self-expression values, summed with the covariance between the two. This rule only applies if both of the paths have a random effect.[3] The maximum likelihood estimate of this product is $\overset{EDU}{\gamma_{10}} * \overset{SEV}{\gamma_{20}} + Cov(\overset{EDU}{\gamma_{10}}, \overset{SEV}{\gamma_{20}}) = -1.427 \times (-0.204) + 0.001 = 0.293^{**}$.[4] Similarly, education has a direct effect on self-expression values ($\overset{SEV}{\gamma_{20}} = -0.204^{***}$), as well as an indirect one, through income. In this case, single-level tracing rules apply, since at least one of the paths, or in our case both education on income and income on self-expression values, is not allowed to vary across countries. Therefore, $\overset{INC}{\gamma_{10}}\,\overset{SEV}{\gamma_{10}} = 0.015^{***}$.

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[2] We also theorized an indirect effect through income, but the results revealed that age does not explain income, at least in our model specification.

[3] The implementation of this formula in various software packages is given by Bauer, Preacher, and Gil (2006).

[4] The standard error–based confidence intervals for this product can be obtained directly through maximum likelihood estimation using Sobel's (1982, 1986) delta method (Bollen, 1987)—not to be confused with the delta parameterization available in *Mplus* for categorical outcomes. In small samples, this estimate may be biased, in which case a bootstrap is preferable (Preacher et al., 2010).

The model we test, though, presents an additional mediation: *GDP*, a Level 2 exogenous covariate, exerts its effect on lower-level variables (like education and self-expression values) both directly and indirectly through its effect on a Level 1 covariate of self-expression values: education. This is a 2 $\rightarrow$ 1 $\rightarrow$ 1 configuration. In the MLM setting, the direct effect of *GDP* on self-expression values is transmitted through the intercept of *SEV*, which is allowed to vary across the countries.

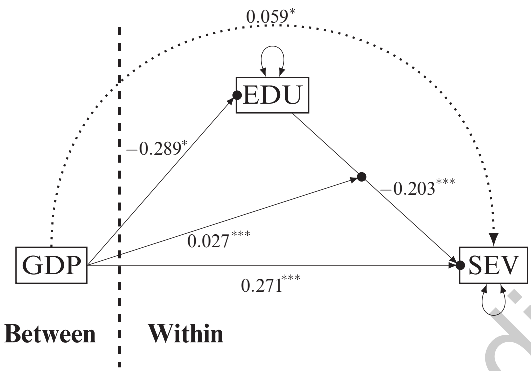
Figure 2.7 extracts just the parts of the model that are of interest when assessing the direct and indirect effect of *GDP* on self-expression values: the slope of *EDU* on *SEV* and how this slope, as well as the intercepts of *EDU* and *SEV*, are affected by *GDP*. Here, the line separating the levels of analysis is now vertical and the mediation is presented in the usual triangular form. At the individual level, the effect of GDP per capita on self-expression values is clearly moderated by educational achievement ($\overset{SEV}{\gamma_{21}} = 0.027^{***}$). At low levels of GDP per capita, the gap between those with high and low education is *larger* than the corresponding gap in countries with high levels of GDP per capita. At the same time, the impact of *GDP* on self-expression values confirms what we discovered in our random intercepts specification: Wealthier countries exhibit higher levels of self-expression values in the citizenry.

Note how, as opposed to the model in Figure 2.3, in Figures 2.5 and 2.7, *GDP* has a significant impact on education ($\overset{EDU}{\gamma_{01}} = -0.289^{*}$). This significant path is what makes the assessment of a potential indirect effect of GDP per capita on self-expression values worthwhile. Once again, just like in single-level SEM, the maximum likelihood estimate of the indirect effect of *GDP* on *SEV* is the product of the two direct paths: from *GDP* to *EDU* and from *EDU* to *SEV*. In this case, this yields a significant positive effect: $\overset{EDU}{\gamma_{01}} \times \overset{SEV}{\gamma_{20}} = -0.289 \times -0.203 = 0.059^{**}$, which is depicted in Figure 2.7 using a dotted arrow.

Moderation

Unlike mediation, *moderation* refers to an instance where a variable alters the strength or direction of the relationship between an exogenous and an endogenous variable (Baron & Kenny, 1986, p. 1174), as is depicted in Figure 2.6b. In this framework, there is a debate on the causal ordering of X, M, and Y. Kraemer et al. (2008; 2002) argue that M must be prior to and uncorrelated with X. For this reason, for example, the same variable could not be a mediator and moderator at the same time, since a mediator M is necessarily directly associated with

Figure 2.7 Moderated Mediation Effect in the Slopes Model



Note: Figure includes only the variables involved in the effect of *GDP* on self-expression values, both directly and through education. The line separating the levels of analysis is now vertical and the mediation is presented in the usual triangular form.

X by means of a causal pathway. Hayes (2013, pp. 399–402), however, demonstrates mathematically how this independence between *M* and *X* is not a requirement, and therefore, the causal link between *X* or *Y* and *M* could be modeled if it makes theoretical and substantive sense. Hayes (2013, pp. 209–210) lists a large number of theories in social sciences that rely on evidence provided by moderation. Petty and Cacioppo's (1986) *elaboration likelihood model*, for example, relies extensively on arguments that imply moderation. Persuasion effects will be stronger depending not only on characteristics of the message (e.g., if it originates with an expert) but also on an individual's motivation and ability to think about the specific topic that the message addresses. In their account, individual motivation moderates the association between message characteristics and persuasion. Solt's (2008) updated version of Goodin and Dryzek's (1980) *relative power theory* sees income inequality as moderating the relationship between income and political participation: In countries with higher inequality, the difference in terms of participation between rich and poor voters is higher than in countries with lower-income inequality.

In the model we test here and which we present in a truncated form in Figure 2.7, GDP per capita has a direct effect on education and self-expression values but also moderates the relationship between education and self-expression values. In countries with high GDP (or, in this case, the natural logarithm of GDP), the negative relationship between education and self-expression values becomes weaker. This represents precisely the type of moderated relationship we have discussed above, which takes place between a Level 2 moderator and a Level 1 relationship. A second mediation relationship is present in Figure 2.5, although not also depicted in Figure 2.7: The impact of age, mediated by education, is now also moderated by *GDP*.[5]

In essence, the random slopes model gives us the ability to test both direct and indirect relationships moderated by higher-level characteristics. In addition to this between-level moderation of within-level relationships, it is possible to assess direct and indirect effects of a higher-level (between-level) phenomenon on a lower-level (within-level) outcome. We have seen from Figure 2.5 how, in the presence of random slopes, it is somewhat difficult to read within-level direct and indirect effects. One has to consult the structure in both the within and the between level. When aiming to assess direct and indirect effects across levels, the situation is quite similar.

In the random slopes model, these sets of relationships become more complex. The first issue that is apparent is that between Figures 2.3 and 2.5, the sign of the direct effect of age on self-expression values has changed. This may not be apparent at first glance in Figure 2.5 as this relationship is now allowed to vary across the countries, making it a latent variable in the between level. Still, this latent's mean expressed on the path going from the triangle to $\overset{SEV}{\beta_{3j}}$ highlights how this relationship is now positive and significant and not negative and significant. The reasons for this change are manifold. First, this model is substantially different and, based on the comparison of fit, better describes the data. Some of the indirect effects are now modeled differently. More important, though, in the second model, an interaction is modeled explicitly. The impact of age on self-expression values, now moderated by *GDP*,

[5] In the interest of clarity, as well as to assist the reader in absorbing these concepts, we have chosen to present these two concepts as separate. In our model, though, the connections between *GDP*, education, and self-expression values represent a fairly typical case of moderated mediation across two levels of analysis: country and individual. In SEM, moderated mediation is typically modeled on a single level of analysis through the inclusion of the product of the moderator and the explanatory factor of interest, which is included as a new variable in the model. This is not the case here. The product variable is not explicitly included as an additional variable; rather, the moderation is modeled as the impact of a between variable on the random effect of a path.

decreases with every doubling of GDP. Considering the range of GDP's logarithm (between 6.99 and 11.23 in this data set), this interaction does not fully account for the change here, but it is true that for the United States, this direct impact (which, we should not forget, is also controlling for indirect impacts) is estimated to be negative.

Centering in MSEM

The reader might be puzzled as to why we did not use a common technique in the multilevel modeling world for easing the interpretation of this interaction: centering. Widely discussed in the multilevel modeling literature (Enders & Tofighi, 2007; Kreft, de Leeuw, & Aiken, 1995; Paccagnella, 2006), centering is also applicable in the case of multilevel structural equation models. To produce a more meaningful interpretation of coefficients, centering can take the form of either group mean or grand mean centering (see Enders & Tofighi, 2007). If *GDP* were grand mean centered, the direct effect of age on self-expression values would be estimated for the average GDP level (as *GDP* would be 0 for the country with the average GDP). Group mean centering of Level 1 variables, on the other hand, changes their interpretation as well as their estimated effect on a Level 1 outcome. This occurs because group mean centering (subtracting the average for each country out of the measured values on an indicator such as age or income) erases all between-country variation in a variable. In the aftermath of this procedure, all that is preserved is relative positions of Level 1 units inside a group. Although the average age in Japan is much higher than the average age in Egypt, after centering, only relative differences between individuals inside a country would be preserved. This produces a clear estimate of a Level 1 variable's effect on a Level 1 outcome, disregarding all dynamics between countries.

In MSEM, centering is complicated by the possibility of statistical associations between indicators measured at various levels of the data hierarchy. MSEM can not only accommodate the standard path modeling situation ($1 \rightarrow 1 \rightarrow 1$), but it can also estimate more intricate pathways of association: $2 \rightarrow 1 \rightarrow 1$, $1 \rightarrow 2 \rightarrow 2$, or even $1 \rightarrow 2 \rightarrow 1$. This added flexibility comes at the cost of greater care when deciding whether a relationship in the pathway should be estimated based on within or between variation.

We begin our discussion by relying on a framework proposed by B. O. Muthén and Asparouhov (2008). Multilevel mediation can be estimated by partitioning the multilevel structural equation model into (1) a measurement model, (2) a "within" structural model, and (3) a "between" structural model.[6] To achieve this separation of the structural model, each Level 1 observed indicator is partitioned into a "within" and a "between" latent component. These are then used to specify the (2) and (3) models: A 2 → 1 component would be estimated using only the "between" latent of the Level 1 indicator, while the continuing 1 → 1 pathway would be estimated using only the "within" latent of the same indicator.

In *Mplus*, these steps are performed automatically as part of the estimation procedure, even if the data contain raw (before centering) versions of the indicators (Preacher et al., 2010, p. 215).[7] The reader should therefore be confident that centering was performed in the analyses we report so far, even though we have not discussed centering until now. Taking the example of the model presented in Figure 2.5, the "within" effect of education on self-expression values is estimated using only "within" variation in education, as *Mplus* has already performed the decomposition into two latents we discuss above. On the other hand, the "between" effect of GDP per capita on the intercepts for education is estimated using only "between" variation in education.

Other software does not automate this partitioning process, which is why the researcher is entrusted to perform it. In such instances, a researcher would construct the group-level latent *configurational* construct (using the terminology introduced by Kozlowski & Klein, 2000) out of the individual-level observed indicators. Following this, the researcher would proceed as in standard MLM by making sure that any 1 → 1 pathway is estimated using the "within" latent and that a 2 → 1 or 1 → 2 pathway, or even the more complicated patterns found in three-level MSEM, is estimated using the "between" (*configurational*) latent.

Summary

This chapter highlighted examples that are possible when path analyses are extended into the multilevel framework. The kind of modeling flexibility demonstrated here is simply not possible using a basic multilevel

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[6] We encourage more advanced readers to consult B. O. Muthén and Asparouhov's original text or, at a minimum, the brief exposition made by Preacher et al. (2010).

[7] See also the original discussion in L. K. Muthén and Muthén (1998–2017, p. 261) related to group mean centering when disentangling "within" and "between" variance components for latent covariate decomposition.

model and, in fact, goes beyond what we have shown so far. For example, it is entirely possible to model full structures of relationships on the between level. The models presented here used a single Level 2 variable, *GDP*, in both the context of moderating Level 1 relationships and as a crucial covariate of a Level 1 phenomenon. But it is possible to include multiple Level 2 variables in the model and not just as exogenous variables. It would be possible to include other country-level phenomena not just as direct predictors of Level 1 outcomes and moderators of Level 1 relationships but also as predictors of Level 2 phenomena, extending the possibility of cross-level mediation structures. Say, for example, *GDP* was the mediator of a Level 2 explanatory factor that had both direct and indirect effect on self-expression values. The number of modeling possibilities seem to be limited only by researchers' theories and available data.

While latent variables have already emerged on Level 2, as varying intercepts and slopes, the real flexibility of SEM, in its single-level form, comes from its ability to include multiple-indicator latent variables. In the following chapter, we present the most basic latent variable SEM, a confirmatory factor model's extension into multilevel modeling, as a stepping-stone to full structural models in the multilevel framework.