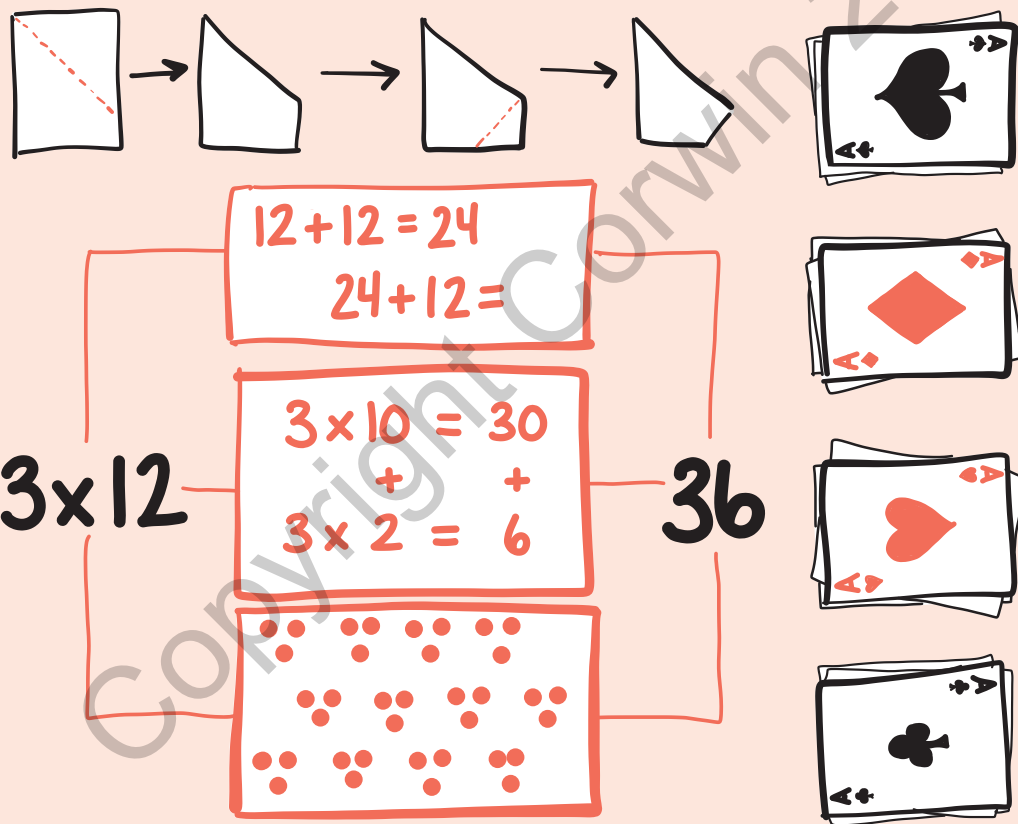




# CHAPTER 1

WHAT TYPES OF TASKS WE USE  
IN A THINKING CLASSROOM

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If we want our students to think, we need to give them something to think about—something that will not only require thinking but will also encourage thinking. In mathematics, this comes in the form of a task, and having the right task is important. So, while the rest of the book will look at the things we can do in our teaching practice to build thinking classrooms, this chapter will look specifically at the tasks around which thinking classrooms are built. By the end of this chapter you will have learned about the different types of tasks that you can use to build a thinking classroom, where to find them, and how to design your own.

If we want our students to think, we need to give them something to think about.



## THE ISSUE

Tasks are inert. To come alive, they need an audience to solve them. So, when I talk with teachers about what makes a good task for building thinking classrooms, I don't talk about what a task is, but rather what a task does. And what a task needs to do is to get students to think. Consider, for example, the following task:

Which is greater, eight or nine?

You may be thinking that this is not a good task. And if this question were posed to Grade 9 students, you would be correct. That is the wrong audience for this task. But if this same question were asked of a four-year-old child, this turns out to be a very good task. The strategies that the child would need to invoke in order to figure this out are both complex and nuanced and would require a lot of thinking to resolve. So, the question is not whether *which is greater* is a good task or not. The question is, what is it good for? And the answer to that question is that it is good for getting students, for whom the relative cardinality and/or positionality of the number symbols have not yet been routinized, to think.

When it comes to talking about tasks that get students to think, the best place to start is with problem solving. From Pólya's (1945) *How to Solve It* to the *NCTM Principles and Standards* (2000), the literature is replete with the benefits of having mathematics students engage in problem solving. Although there are arguments about the exact processes involved and the exact competencies required, there is universal agreement that problem solving is what we do when we don't know what to do. That is, problem solving is not the precise application of a known procedure. It is not the implementation of a taught algorithm. And it

Problem solving is what we do when we don't know what to do.

is not the smooth execution of a formula. Problem solving is a messy, non-linear, and idiosyncratic process. Students will get stuck. They will think. And they will get unstuck. And when they do, they will learn—they will learn about mathematics, they will learn about themselves, and they will learn how to think.

**Good problem-solving tasks require students to get stuck and then to think, to experiment, to try and to fail, and to apply their knowledge in novel ways in order to get unstuck.**

As with good tasks for building thinking classrooms, what makes a good problem-solving task is based on what it does—or rather, what it requires students to do to solve it. Good problem-solving tasks require students to get stuck and then to think, to experiment, to try and to fail, and to apply their knowledge in novel ways in order to get unstuck. The *cats and rats* problem in the introduction is a good example of such a task. Knowledge of fractions and ratios is necessary, but far from sufficient, to solve this problem. Yet, no other mathematical content knowledge is needed. To solve it—to get unstuck—we need to think about the problem differently than we usually think about equivalent fractions or common ratios. We need to come to the realization that if six cats kill six rats in six minutes, then either six cats will kill one rat in one minute, or

one cat will kill one rat in six minutes. How a student gets to this realization is problem solving.



Problem-solving tasks are often called non-routine tasks because they require students to invoke their knowledge in ways that have not been routinized. Once routinization happens, students are mimicking rather than thinking—or as Lithner (2008) calls it, being imitative rather than creative. Good problem-solving tasks are also rich tasks in that they require students to draw on a rich diversity of mathematical knowledge and to put this knowledge together in different ways in order to solve the problem. They are also called rich because solving these problems leads to engagement with a rich and diverse cross section of mathematics. Regardless of how they are referred to, what makes a task a good problem-solving task is not what it is, but what it does. And what they do is make students think.

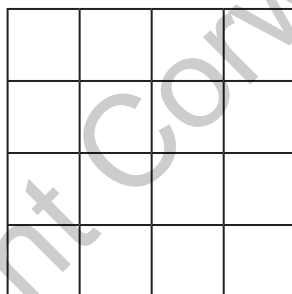
My early research into building a thinking classroom was very much focused on tasks. Despite my experiences in Jane's class, I still believed that the best way to get students to think was to give them a task that would motivate, even necessitate, them to think. For this reason, I spent a lot of time searching for and designing tasks that would do

just that. What emerged from these efforts was a collection of what I started out calling *highly engaging thinking* tasks. To this collection I also added a lot of mathematical *card tricks* and developed a genre of real-world problem-solving tasks that I called *numeracy tasks*.

Let's take a closer look at each of these three kinds of tasks:

1. **HIGHLY ENGAGING THINKING TASKS** are so engaging, so interesting, that people cannot resist thinking. They have broad appeal and can be used across a wide range of grades, with some being able to be used all the way from Grade 4 up to calculus and beyond. At first, I thought they were rare—so rare, that for a long time I didn't know if they really existed. And then I found one. And then another. And then several. Now I realize they are plentiful if you know where to look. Here are four examples of such tasks, organized by grade band:

- **PRIMARY:** How many squares are in the image below?



- **INTERMEDIATE:** I buy a video game for \$10. I then sell it for \$20. I buy it back for \$30. Finally, I sell it again for \$40. How much money did I make or lose?
- **MIDDLE SCHOOL:** I have a four-minute egg timer and a seven-minute egg timer—the kind that you turn over and let the sand run through. Can I use these to cook a nine-minute egg? If so, how long will someone have to wait for their egg?
- **HIGH SCHOOL:** An eccentric woman has booked three adjacent and adjoining hotel rooms. When she checks in, she tells the receptionist that if he needs her, she will always be in the room next door to the room she was in the night before. The receptionist thinks nothing of this until an hour later when he realizes that her credit

card has been declined, and he must now go find her. The problem is that he is very busy and only has time to knock on one door per day. How many days does he need to guarantee that he finds her? What if it were four rooms? Five rooms? What if it were 17 rooms, and she is checked in for 30 days—can he find her before she leaves?

I will share more of these types of highly engaging thinking tasks throughout the book, beginning with some at the end of Chapter 3.

2. **CARD TRICKS** have the same qualities as highly engaging thinking tasks—they are highly engaging situated tasks that draw students in and entice them to think. It turns out that there are a lot of card tricks that are both built on and can be explained by mathematics. These were the ones I was interested in. What I was not interested in were card tricks that relied on sleight of hand. I wanted students to engage with the magic of mathematics, not the magic of my hands. Video 1.1 shows an example of one of these tasks. If you are interested in these kinds of card tricks, you can find a collection of them on my website (<http://www.peterliljedahl.com/teachers/card-tricks>).



#### VIDEO 1.1

If you are interested in these kinds of card tricks you can find a collection of them on my website (<http://www.peterliljedahl.com/teachers/card-tricks>).



Source: Youtube video via [peterliljedahl.com](http://peterliljedahl.com)

3. **NUMERACY TASKS** are tasks that are based not only on reality, but on the reality that is relative to students' lives. From cell phones to entertainment to sports, these tasks are built up specifically to engage students in rich tasks wherein they have to negotiate the ambiguity inherent in real-life experiences. For example,

## SKI TRIP FUNDRAISER

The ski club is finally going skiing. Each person tried their best to raise money for their trip. Below is a chart that shows how much money each person raised, and their individual cost, depending on whether they need rentals or lessons. All of the money raised must be applied to the cost of the trip, and every person must go on the trip, even if it means that they may have to put in their own money to do it. Have they raised enough? If not, who needs to pay, and how much do they need to pay?

Name	Amount Raised	Rental Cost	Lift Ticket	Lesson Cost
Alex	75	20	40	40
Hilary	125	10	40	40
Danica	50	30	40	0
Kevin	10	40	40	40
Jane	25	0	40	0
Ramona	10	0	40	40
Terry	38	30	40	0
Steve	22	40	40	40
Sonia	200	20	40	0
Kate	60	25	40	0

More examples of these tasks, and how they are made, can be also be found on my website (<http://www.peterliljedahl.com/teachers/numeracy-tasks>).

All three of these types of tasks provide engaging contexts that draw students in and entice them to think. Therefore, these tasks are useful in building thinking classrooms. Aside from context, all these tasks also have easy entry points (low floor) and evolving complexity (high ceiling), and they drive students to want to talk and to collaborate.

Whereas the inherent ambiguity of numeracy tasks makes them truly open ended—with some having as many as 200 viable and defensible solutions—the highly engaging thinking tasks and card tricks usually have only one final answer. However, they allow for multiple approaches to get to that one answer and, hence, have an open-middle structure.

### Low-Floor Task:

Task with a threshold that allows any and all learners to find a point of entry, or access, and then engage within their level of comfort.

### High-Ceiling Task:

Tasks that have ambiguity and/or room for extensions such that students can engage with the evolving complexity of the task.

### Open-Middle:

A problem structure where a task has a single final correct answer, but in which there are multiple possible correct ways to approach and solve the problem.



## THE PROBLEM

Aside from being rich and engaging tasks with the ability to get students to think, these aforementioned tasks share another quality—they are, for the most part, all non-curricular tasks.

That is, very few of these tasks require mathematics that map nicely onto a list of outcomes or standards in a specific school curriculum. Consider, for example, the difference between two tasks that can be used with Grade 8 students: the *True or False* card trick in Video 1.1 and a task that asks students to add two proper fractions with different denominators. The *True or False* task is clearly mathematical in nature; the solution to it requires that students attend to the position of the target card, the patterns in the cardinality of the number of letters in certain words, and the role that reversing order plays—none of which is an outcome in a Grade 8 curriculum. On the other hand, asking students to add two fractions with different denominators requires them to understand that a common denominator is needed, be able to find the lowest common denominator, add fractions, and potentially be able to reduce a fraction—all of which are outcomes in some Grade 8 curricula. So, whereas both tasks are mathematical in nature, the *True or False* card trick is non-curricular, while the adding-fractions question is curricular.

**Non-curricular Task:** A task that is clearly mathematical in nature but does not map well to the outcomes or standards specified in the curriculum for the class in which it is used.

Even if there is a rich task that maps nicely to the curriculum you are teaching, it only maps to curricular outcomes if students happen to solve the problem using concepts and skills from their current curriculum. This is the nature of open-middle and open-ended tasks. Such tasks invite students to think for themselves. And when students begin to think for themselves, a lot of unpredictable things can happen. If your goal is only to get students to think, then this is not a problem. If your goal is to use a rich task to, for example, get students to think about division of fractions, then this can be a problem. Of 30 students, only a handful may choose a solution path that follows the lines of curriculum you were hoping a rich task would touch on. The rest may choose to use repeated subtraction, repeated addition, or a type of logic that makes unnecessary the need to think about fractions at all. Depending on the grade you are teaching, these solution paths, although not achieving what you were hoping for, may still touch on topics from your curriculum. More often, however, this is not the case.

If, in reaction to this, we try to force a more predictable curriculum mapping by artificially constraining tasks, before long we have

reduced what was once a rich task to the type of word problem we often see in mathematics textbooks:

Camille went to the store to buy eggs, milk, and cheese.  
Eggs cost \$3.50, milk costs \$2.00, and cheese costs \$4.00.  
How much money did Camille need?

Word problems, like rich tasks, require the student to decode what is being asked. However, once a word problem is decoded, the mathematics is often trivial, procedural, and analogous to the mathematics that was taught that day. This is not true of rich problem-solving tasks. In a rich task, once the language has been decoded, the mathematics that is needed to solve it is neither trivial nor procedural. Basically, in rich tasks the problem is in the mathematics, and in word problems the problem is in the words—this is maybe why they are called word problems.

Once a word problem is decoded, the mathematics is often trivial, procedural, and analogous to the mathematics that was taught that day. This is not true of rich problem-solving tasks.

Whereas rich tasks get students to think at the expense of meeting curriculum goals, word problems more predictably and reliably push students to use specific bits of learned knowledge—but often at the expense of engagement and the thinking that we need to foster in our students. So, how then do we move forward from this reality?



## TOWARD A THINKING CLASSROOM

One way forward, although seemingly unrealistic, is to stop worrying about curriculum. My earliest efforts to build thinking classrooms did just this. Rather than think about curriculum, I was only concerned with getting students to think. This is not to say that I was naïve about the lived reality of classroom teachers and the persistent and ubiquitous nature of curriculum. Rather, it is just that I needed to start somewhere. Before I could even begin to think about how to get students to think about curriculum, I needed to get students to think.

This proved to be surprisingly easy. Once we shed the burden of curriculum, it turns out that there are a huge number of resources available to us that are effective for getting students to think. From problems of the day to brainteasers, the internet is full of resources that are engaging and thought provoking. Some of these, it can be



argued, address curriculum—but, again, only for those students who follow a particular solution path.

Students, as it turns out, want to think—and think deeply. My early efforts to build thinking classrooms through the use of highly engaging thinking tasks, card tricks, and numeracy tasks—and my cavalier attitude about curriculum—were actually hugely successful. Successful to the point where I could give a teacher a set of three tasks and, without any other changes, could dramatically increase both the number of students who were thinking and the number of minutes that were spent thinking. On top of that, students were enjoying and looking forward to mathematics and the next task, their self-confidence and self-efficacy increased, and they became better mathematical thinkers.

Students, as it turns out, want to think—and think deeply.



**Figure 1.1** Students in an elementary classroom engage in a thinking task. Source: FatCamera/iStock.com

The trick was to maintain the positive effect, and positive affect, while turning our attention back to the reality of curriculum. To do this, I had one thread to follow—the thread that comes from the understanding that problem solving is what we do when we don't know what to do. Curriculum tasks are typically the exact opposite of this. Curriculum tasks are often what students do when they know what to do—*after* they have been shown how. Asking a high school student to factor  $x^2 - 5x - 14$  or an elementary student to solve  $3.1 + 5.2$  after they have been shown how promotes mimicking, not thinking. My observation of those initial 40 classrooms showed that this is exactly when and how curriculum tasks were most often used.

Having said that, it turns out that both of these questions are excellent thinking questions—if they are asked *before* the students have been shown how to answer them. Herein lay the root of how to get students to think while at the same time addressing grade-specific curriculum. For example, let's look more closely at the factoring quadratic task and how that question can be presented without first teaching students how to do it.

Asking a high school student to factor  $x^2 - 5x - 14$  or an elementary student to solve  $3.1 + 5.2$  after they have been shown how promotes mimicking, not thinking.

**Teacher** Let's start with a bit of review. How would I expand  $(x + 2)(x + 3)$ ?

[Teacher writes on the board  $(x + 2)(x + 3) =$ ]

**Students**  $x^2 + 5x + 6$ .

[Teacher writes on the board  $(x + 2)(x + 3) = x^2 + 5x + 6$ ]

**Teacher** OK. So what if my answer were  $x^2 + 7x + 6$ ? What would the question be?

[Teacher writes on the board ( ) ( ) =  $x^2 + 7x + 6$  right underneath the previous line.]

For adding decimals, the question could be posed in a similar fashion.

**Teacher** Let's start with a brief review. Can someone tell the class what 3.1 means?

**Student** This is a number that is bigger than 3 but less than 4.

**Teacher** Is it closer to 3 or 4?

**Student** It is closer to 3.

**Teacher** OK. And what is 5.2?

**Student** It is a number between 5 and 6 that is closer to 5.

**Teacher** OK. If I add 3.1 and 5.2, what two whole numbers is the answer between, and which number is it closer to? What would the answer be?

Even counting at the primary level can be turned into a thinking task.

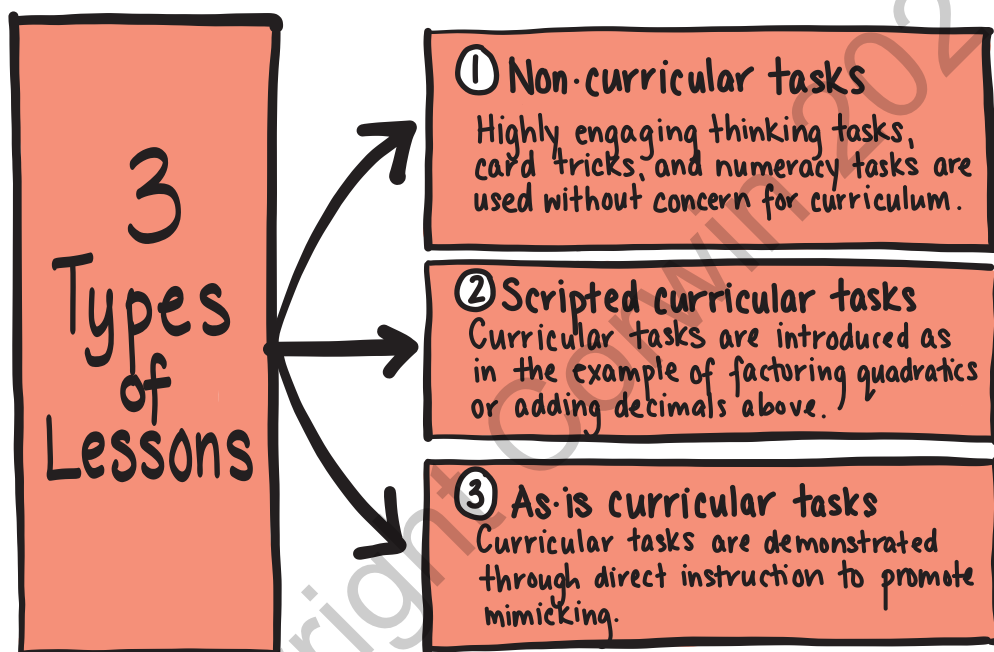
**Teacher** Let's all count together up to 20.

**Students** 1, 2, 3, 4, 5, . . . , 20.

**Teacher** Ok. What if we start at 14? What are the three numbers that come after 14? What are the three numbers that come before 14?

These scripts are similar in that they begin by asking a question about prior knowledge, then they ask a question that is an extension of that prior knowledge, and they ask students to do something without telling them how. And, as such, they require students to think, not only in general, but also about particular curriculum. It turns out that almost any curriculum tasks can be turned from a mimicking task to a thinking task by following this same formulation—begin by asking a question that is review of prior knowledge; then ask a question that is an extension of that prior knowledge.

In my research, I compared three types of lessons (Figure 1.2).



**Figure 1.2** Three types of lessons.

There were big differences between how students performed in these types of lessons. Although the first two lesson types were both designed around tasks to get students to think, the lesson that was designed around non-curricular tasks (Type 1) got many more students to think than the lesson scripted to get students to think about curriculum (Type 2). Simply turning a standard curricular task into a thinking task was not enough to get all the students thinking.

Similarly, whereas the second and third types of lessons are both built around curriculum tasks, the lesson where direct instruction was used (Type 3) allowed more students to successfully complete the task at hand. This is not surprising, as mimicking can be an effective strategy that may allow students to be successful in the short term.

But, as mentioned in the introduction, mimicking is not thinking and therefore not learning. Naturally, there were also students who were successful on the rescripted curricular thinking task (Type 2). There were just fewer than in the direct instruction lesson.

However, an interesting thing happened when three lessons using non-curricular tasks (Type 1) *preceded* students' exposure to the scripted curricular thinking task (Type 2)—the number of students who successfully completed the scripted tasks (Type 2) surpassed the number who were successful in the mimicking lessons (Type 3). In other words, students can be successful at these types of scripted thinking tasks, even more successful than in lessons designed to promote mimicking, if their willingness to think is *first* primed with the use of good non-curricular tasks. This makes sense. Type 1 tasks are more likely to engage students with their rich and interesting contexts and propel them into thinking than a task asking them to think about factoring quadratics, adding decimals, or counting. But once the thinking starts, it becomes an end unto itself, and students are not only more willing to think but they want to think. The non-curricular tasks (Type 1), in this regard, served as a primer for—and thus made room for—the more curriculum-driven scripted thinking tasks (Type 2).

Once the thinking starts, it becomes an end unto itself, and students are not only more willing to think but they want to think.

Further investigation showed that although three lessons of non-curricular tasks (Type 1) was enough to prime many classes, in some cases as many as five lessons were needed before the dispositions of the students shifted enough to allow them to be successful at scripted curricular thinking tasks. This investigation revealed that, in almost every situation, the teacher was able to predict when the class was ready to shift their thinking toward curricular thinking tasks.

**Lucy** I don't know why, but they just seemed ready. There was no more whining, and the kids came into class excited about seeing the problem they would work on that day.

The key was, however, that in the transition from a non-curricular task (Type 1) to a curriculum thinking task (Type 2), nothing else changed. The teacher posed the task as a challenge—as a problem to solve—without any big declarations that now we are going to start doing curricular tasks in a different way.

This is not to say that all students were successful or that all students were willing to think. Far from it. Simply turning a basic curriculum

task into a thinking task does not mean students are automatically going to think. More things need to change in the lesson if thinking is to be built and sustained over time, and that is what the rest of this book is about. However, these results show that to get students thinking *about* curriculum tasks, they need to first be primed to do so using non-curricular tasks. Nothing in my research has shown a way to avoid this. You have to go slow to go fast.

In Chapter 9, I will discuss much more about how to build a sequence of scripted curricular thinking tasks (Type 2) that follow on the heels of the aforementioned engaging non-curricular tasks (Type 1) and allow students to effectively think their ways through large amounts of curriculum quickly. For now, however, it is sufficient to say that the goal of this book is not to get students to think about engaging non-curricular tasks day in and day out—that turns out to be rather easy. Rather, the goal is to get more of your students thinking, and thinking for longer periods of time, within the context of curriculum.

## FAQ

**Q** In this chapter, as well as the introduction, mimicking is portrayed as something bad. Isn't mimicking a good starting point for students before moving onto thinking tasks?

**A** The question is not whether mimicking is good or bad. The question is, what is mimicking good or bad for? Mimicking is very good at teaching students how to replicate routines—the routine for factoring quadratics, adding decimals, dividing fractions, et cetera. So good, in fact, that once students start to have success with mimicking, they don't want to stop. Mimicking is an addiction that is easily acquired at lower grades and difficult to give up at higher grades. You may have seen this when trying to explain a difficult concept and some of your students are asking you to “just show us how to do it.” The problem is that mimicking is only an effective strategy when the number of routines to memorize is small. As the student moves up in grades, the number of routines per topic increases, until this becomes an unmanageable and ineffective strategy. Yet students who have had success with it in the past are resistant to abandoning it. Furthermore, mimicking tends to create short-term success without the long-term learning that allows students to make connections with other topics in the same and subsequent grades. So they do not develop the web of connections that helps them understand mathematics.

Mimicking is bad because it displaces thinking. Mimicking happens not alongside, but instead of, thinking. Likewise, mimicking is not a precursor to thinking. Mimicking requires less energy and less effort than thinking, and once the mimicking has begun, it is difficult to ask students to shift their attention to something that takes more time, more energy, and more effort. Our research on studenting and homework showed that only 20% of students who mimicked at the beginning of their homework assignment were even willing to attempt questions for which they did not have an analogous worked example and that would require them to think. And of those, only half were able to complete a question for which they did not have an analogous example in their notes.

**Q** I don't have time to give up three to five days of my school year to do non-curricular tasks. Can't I just jump right in with curriculum thinking tasks?

**A** Starting to build thinking classrooms with non-curricular tasks is imperative. As already mentioned, their use dramatically increases your students' success with scripted curriculum thinking tasks when you transition to those types of tasks. How it does this has not been mentioned, however. Well selected non-curriculum tasks, with their engaging contexts, propel students to want to begin to think. They create situations where every student gets stuck, which makes stuck an expected, safe, and socially acceptable state to be in. In essence, these tasks make it safe to fail and keep trying. And through these struggles, students begin to build confidence in their teacher's confidence in them. All of these qualities are easier to build inside of highly engaging non-curricular tasks and are necessary when we transition students to curricular thinking tasks.

This is not to say that these same qualities can't be built inside of curricular thinking tasks, but it is harder, takes longer, and will only work well with a few students. Curricular tasks are too familiar to signal that something has changed, and thereby are less likely to prompt a change in behavior.

**Q** If non-curricular tasks—especially highly engaging thinking tasks—are so good at engaging students, why don't we just teach all of mathematics that way? There must be a collection of tasks, the whole of which will cover an entire curriculum.

**A** This is a bold approach, which has been proven to work. This is the essence of Jo Boaler's early research at Phoenix Park (Boaler

2002). Further, Maria Kerkhoff (2018) showed that after doing just 18 rich tasks over the course of 18 classes, the student who she was studying encountered almost all of the curriculum outcomes for her grade, along with numerous curriculum outcomes from previous and future grades. In essence, if we just get students thinking about lots of different problems, the curriculum outcomes will eventually be covered, irrespective of which solution paths students follow. This is the approach a group of mathematics educators in Alberta took. They have created collections of tasks for Grades 2, 3, and 8, which allow them to cover all of the curriculum. You can access these collections by going to Alicia Burdess's website (<http://www.aliciaburdess.com/teaching-through-problem-solving.html>).

The problem is that such a move takes a lot of faith on the part of the teacher. And this faith is quickly eroded if there are set dates by which students must have learned certain concepts. The other issue is that the higher the students get in the grades, the more difficult it becomes to find collections of non-curricular highly engaging thinking tasks that will, in their entirety, cover curriculum—the more abstract mathematics gets, the more difficult it becomes (not impossible) to create such resources.

**Q** Even if I want to use curricular thinking tasks, it will take so much longer to have students think their way to solutions than if I just show them. How will I find the time for that?

**A** There are a lot of aspects of time that came out in the research. First and foremost is the time it takes before students are given an opportunity to answer a question on their own. In lessons designed around having students mimic (Type 3), this opportunity does not occur until 15–35 minutes into the lesson. When using thinking curricular tasks, this happens in a fraction of that time. Looking back at the three sample scripts in this chapter, you will notice they are all brief. Very brief. I will discuss more in Chapter 6 how important this is. For now, however, it is enough to say that when relying on previous knowledge to prompt thinking, these types of scripts will always be brief.

The second aspect of time is how long it takes students to solve a task when asked to think versus when they are asked to mimic. In each of the example scripts, not only is the set-up quicker, the students tend to come to an answer more quickly. This may not be true the first time you design a curricular thinking script, but it goes faster and faster the more adept the students become at thinking.

Finally, my research shows that when curricular thinking tasks are combined with the other 13 practices, students move through a lot of content very quickly. The script for factoring quadratics, for example, when used in a fully implemented thinking classroom context, will cover the entire unit on factoring quadratics in 40–70 minutes. Adding and subtracting decimals takes less. I will discuss this more in Chapter 9. For now, however, it is sufficient to say that yes, it will take more time in the beginning, but you will earn all that back as your classroom becomes a thinking classroom.

**Q** Can students really solve curricular thinking tasks (Type 2) without first being shown how to do them?

**A** Yes. Even when these tasks were introduced on their own, students who were willing to think were generally successful at solving them. But not everyone was willing to think. Using highly engaging non-curricular tasks as a precursor to the curriculum thinking tasks increased dramatically the number of students who were willing to think while at the same time increasing the amount of time that all students were willing to think for—both of which will lead to more students being successful at solving the tasks.

**Q** For a curricular task to generate thinking, it should be asked before students have been shown how to solve it. Does this mean the task should come right at the beginning of the lesson?

**A** Yes. In Chapter 6 I will more thoroughly discuss how important this turned out to be. In the meantime, suffice it to say that thinking tasks should be asked in the first five minutes from the time you begin the lesson.

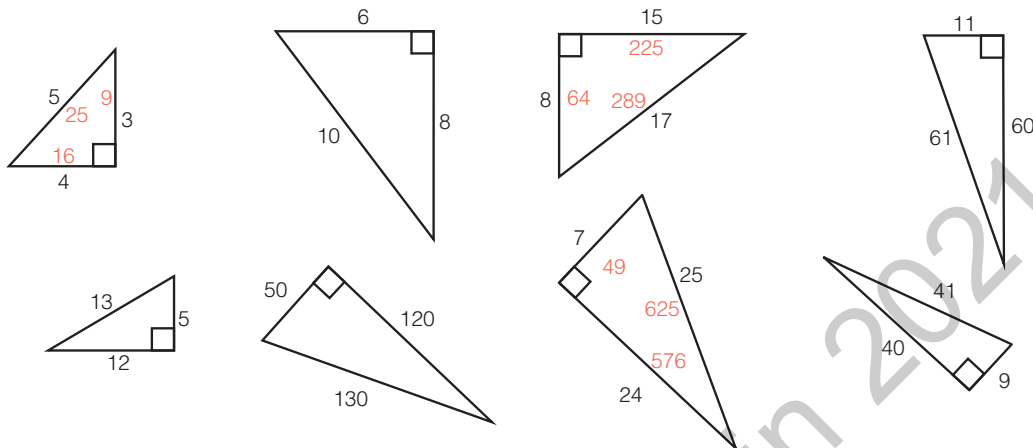
**Q** Each of the examples in this chapter drew on prior knowledge. What does it look like when we are starting with an entirely new topic, a topic for which the students have no prior knowledge?

**A** Curriculum is inherently spiraled. For this reason, it is seldom the case that students have no prior knowledge at all. In the rare cases where it is true, however, you can, if you wish, just tell the students something. But you still only have five minutes before you should ask them a thinking question. Take for example, the introduction of the Pythagorean Theorem. I offer two different scripts that can be used, the first of which relies on pattern spotting.



**Teacher** I am handing out a sheet with eight different triangles [see Figure 1.3], each with all its side lengths indicated. What sorts of patterns do you notice?

**Figure 1.3** Pythagoras sheet.



A sheet structured as in Figure 1.3 would allow students to notice that all the triangles are right triangles. They may also notice that some of the triangles are proportional to each other. They may notice that the extra numbers on some of the triangles are the squares of the sides. Finally, they may notice that there is a relationship among these square numbers.

The second script involves a more direct approach.

**Teacher** If you look at the three triangles I have drawn here, you will notice that they are all right triangles. All right triangles share the property that the sum of the squares of the shorter two sides equals the square of the longer side. This is called the Pythagorean Theorem, and it is written as  $a^2 + b^2 = c^2$ , where  $a$  and  $b$  are the lengths of the shorter sides, and  $c$  is the length of the longer side. For example, we see that in the first triangle  $3^2 + 4^2 = 5^2$ . In the second triangle we see that  $5^2 + 12^2 = 13^2$ . Knowing this, consider this third triangle, where the shorter two sides are 8 and 15. What must the longer side length be?

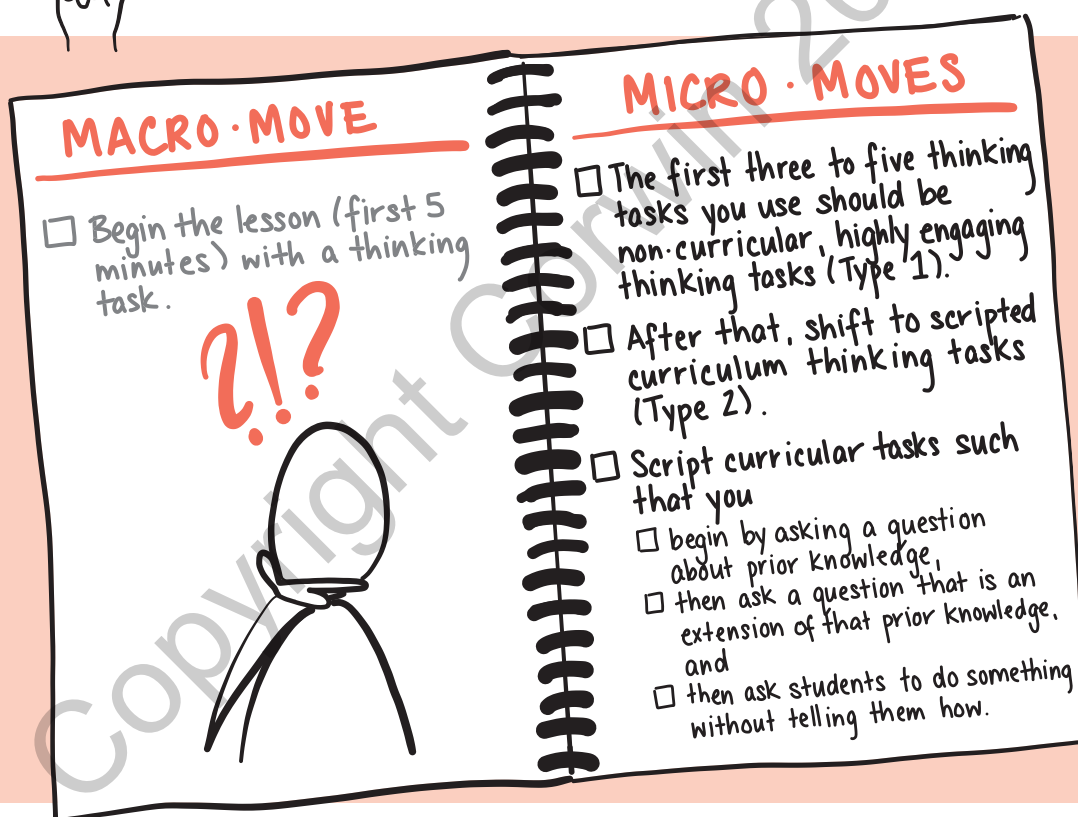
Although very different in approach, both of these scripts have students doing a question they have not been shown how to do in the first five minutes. The first script promotes pattern spotting, while the second approach asks them to apply a known property. Regardless, they are going to have to think their way forward.

**Q** So, I run the script, and the students successfully answer the thinking question I pose. What do I do next?

**A** You ask a similar but more difficult question. All of Chapter 9 is about this, but for now just ask progressively harder questions. For example, in the second script above, you may ask the students to answer a question where the two shorter sides are 3.4 and 5.2 units long. Then ask them to answer a question where they are given the lengths of the longest and one of the shorter sides, et cetera.



## SUMMARY





## QUESTIONS TO THINK ABOUT

1. What are some of the things in this chapter that immediately feel correct?
2. In this chapter you read about the negative consequences of mimicking. Can you think of any positive benefits? If so, do these positive benefits outweigh the negative consequences?
3. The introduction mentioned that almost all students who mimic express that they thought this is what they were meant to be doing. This chapter shares that one of the ways in which students come to this conclusion is by having their teachers show them how to do something before asking them to try it on their own. What other ways may we be communicating that mimicking is what we want students to do—even if that is not what we want?
4. You have read in this chapter that curriculum is inherently spiraled and, therefore, there are very few examples where you would introduce a topic for which students have no prior knowledge upon which such a script can be built. Can you think of some examples of such situations in your curriculum? If you can, is there really no prior knowledge that can be drawn on?
5. In this chapter it was shown that students perform better on scripted curricular tasks if they have first experienced three to five classes of working on highly engaging non-curricular tasks. How do you feel about giving up this time? What are the barriers for you to do this? What do you stand to gain? What do you stand to lose?
6. What are some of the challenges you anticipate you will experience in implementing the strategies suggested in this chapter? What are some of the ways to overcome these?

## TRY THIS

As mentioned in the introduction, the ideas in the first three chapters are best implemented together. Of course, you can ignore this and implement the ideas in this chapter right now. If you are doing this, remember to start with three to five non-curricular tasks and to get students doing these in the first five minutes. If, however, you are going to heed the advice and wait until the end of Chapter 3 to try anything with your students, then this is the time to create some scripts in preparation for this.

This chapter included three examples (counting, adding decimals, and factoring quadratics) of how to script the introduction of a task so that you can ask students to think their way through a problem without first showing them how to do it. These examples are all predicated on the idea of asking the students a question about prior knowledge, and then asking a question that is an extension of that prior knowledge. Consider some topics you have recently taught or are about to teach, and create some scripts for these topics.