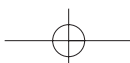
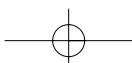
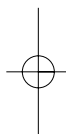
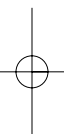
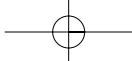


Children's Mathematics





Children's Mathematics

Making Marks, Making Meaning

Second Edition

Elizabeth Carruthers

and

Maulfry Worthington

 **SAGE Publications**
London • Thousand Oaks • New Delhi

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First published 2006

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Paul Chapman Publishing
A SAGE Publications Company
1 Oliver's Yard
London EC1Y 1SP

SAGE Publications Inc
2455 Teller Road
Thousand Oaks, California 91320

SAGE Publications India Pvt Ltd
B-42, Panchsheel Enclave
Post Box 4109
New Delhi 110 017

Library of Congress Control Number: 2006923703

A catalogue record for this book is available from the British Library

ISBN 10 1-4129-2282-8 ISBN 13 978-1-4129-2282-1
ISBN 10 1-4129-2283-6 ISBN 13 978-1-4129-2283-8 (pbk)

Typeset by Dorwyn, Wells, Somerset
Printed in Great Britain by T.J. International, Padstow, Cornwall
Printed on paper from sustainable resources

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About the Authors

Elizabeth Carruthers and **Maulfry Worthington** have each taught in the full 3–8 year age range for over 25 years. Early in their careers both developed incurable cases of curiosity and enthusiasm in Early Years education which fails to diminish. They have carried out extensive research in key aspects of Early Years education, with a particular focus on the development of children’s mathematical graphics from birth – eight years. Publications include articles, papers and chapters on the development of mathematical understanding.

Elizabeth Carruthers is presently head teacher of the Redcliffe Integrated Children’s Centre in Bristol. She has recently worked within an Early Years Advisory Service in a local authority and as a National Numeracy Consultant. Elizabeth has been a mentor with the Effective Early Learning Project (EEL) and has lectured on Early Childhood courses. She has taught and studied in the United States and is currently working on her doctorate researching mathematical graphics and pedagogical approaches. Elizabeth is an advocate for the rights of teenage cancer patients and a supporter of the Teenage Cancer Trust.

Maulfry Worthington is engaged in research for her doctorate on multi-modality within children’s mathematical graphics (Free University, Amsterdam): she also works as an independent Early Years consultant. Maulfry has worked as a National Numeracy Consultant and has lectured in Initial Teacher Education on Primary and Early Years mathematics, Early Years pedagogy and Early Years literacy. She has also worked at the National College for School Leaders as an e-learning facilitator on a number of Early Years online communities and programmes.

Maulfry and Elizabeth are Founders of the international **Children’s Mathematics Network**, established in 2003, described on their website as:

‘an international, non-profit-making organization for teachers, practitioners, students, researchers and teacher educators working with children in the birth–8 year age range. It is a grassroots network, with children and teachers at the heart of it and focuses on children’s mathematical graphics and the meanings children make.

Early 'written' mathematics is explored within the context of visual representation including drawing; early (emergent) writing; schemas; play; thinking; creativity and multi-modal meanings. Our work is based on extensive, evidence-based research with children, teachers and families and within the context of homes, nurseries and schools. We advocate a spirit of freedom and creativity for teachers and more importantly, the freedom for children to explore their own meanings in creative ways. Our aim is to hear the voice of the child.

(See the website at www.childrens-mathematics.net.)

Elizabeth and **Maulfry** are winners of several national awards for their work on mathematical graphics with children and with teachers including TACTYC's 2003 Jenefer Joseph Award for the 'Creative Arts in the Early Years' (3–8), and were shortlisted for Becta's ICT in Practice Award in the 'Innovation and Change' category, 2004.

Dedication

We dedicate this book to our own creative children: Mhairi, Sovay, Laura and Louise, and to the memory of two strong women – our mothers, Elizabeth Gillon Carruthers and Muriel Marianne Worthington.

Acknowledgements

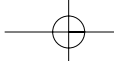
We should like to pay tribute to all the adults and children who contributed to our thinking about children's mathematics.

Our sincere thanks go in particular to Chris Athey who, through her writing, really helped us observe and understand young children's thinking and cognitive behaviour, and also to John Matthews, whose research into children's early marks and drawing has helped us gain further insights in our own work. It was our close analysis of children's mathematical graphics that alerted us to the significance of their marks. It is the meaning in their mathematical marks that enables children to make connections between their own mathematics and abstract mathematical symbolism.

Our thanks go to the other members of the Emergent Mathematics Teachers group, especially to Mary Wilkinson who founded the group and who believed in the importance of teachers writing – for teachers. Our thanks to all the brilliant women teachers in the group who together shared excitement in mathematics education through numerous discussions: Petrie Murchison, Alison Meechan, Alison Kenney, Bernie Davis, Wendy Lancaster, Chryssa Turner, Sue Malloy, Maggie Reeves, Robyn Connett and Julie Humphries.

We would like to thank the staff and children of the following Early Years settings for allowing us to include samples of children's mathematics: Chestnut Avenue Nursery; Littleham Nursery (Fiona Priest) and Walter Daw Nursery. Ide First School, Exeter (Maggie Skeet, Barbara Haddon and Edwina Hill); Stoke Hill First School, Exeter (Steve Greenhaigh); St Nicholas' Combined School, Exeter; Honeywell Infants School, London (Karen Pearson); Weston Mill Primary School, Plymouth (Ann Williams); Hooe Primary School, Plymouth; Bramble Hedge Pre-School, Plymouth (Julie Mills); Uffculm Primary School, Devon; Willand Primary School, Devon; Redhills Combined School, Exeter (Petrie Murchison) St David's First School, Exeter (Wendy Lancaster); The Colleges Children's Centre (Rosie Lesik) and The Cambridgeshire Children's Centres Mathematics Learning Network which includes Histon, Colleges, Brunswick, The Fields, Homerton and, Huntingdon Children's Centres; and to Louise Glovers at Robert Owen Children's Centre, Greenwich.

Our special thanks go to the following people who influenced our thinking over the years and empowered us through their unstinting support: Maggie Skeet; Karen

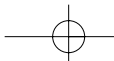
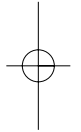
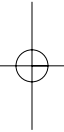


Pearson; Sheila and Michael Rowberry; Heather Tozer; Petrie Murchison; Jean-Anne Clyde, University of Louisville; and the Plymouth Numeracy Team – Chris Clarke, Kathy Jarrett, Liz Walmsley and Rob Pyner.

Thanks go to our families including Steve Worthington and Jane Mulkewich and to all of our friends for their tremendous encouragement. Above all, special thanks must go to the children who have helped us understand, and also to Tom Bass for his encouragement and for making dinners when deadlines loomed.

Cover photograph

The photograph shows a child playing outside at the Robert Owen Children's Centre in London (see p. 166). His teacher Louise Glovers was a member of 'Project 2003': during the year we supported teachers from Early Excellence Centres throughout England as they explored and developed their pedagogy in mathematical graphics, through face-to-face and online discussions.



Foreword

John Matthews

This is one of the most important books on emergent mathematical thought in infancy and early childhood ever written.

Those of us who have devoted our lifetimes attempting to understand the origin and development of expressive, representational and symbolic thought in infancy and childhood, and how best to support it, quickly came to realise that the beginnings of linguistic and mathematical thought are embedded in rather commonplace actions and drawings made by the infant and young child.

Developmentally, these beginnings are of the most profound importance. They form the child's introduction to semiotic systems without which her life in the symbol-rich society of humans will be dangerous if not impossible.

Tragically, these crucial beginnings of expressive, representational and symbolic thought are often discounted completely and receive little or no support from the pedagogical environment.

Why is this? It is because, if these actions are glanced at cursorily, they appear trivial, meaningless and sometimes even as a threat to social control. Children's emergent semiotic understandings are often expressed in free-flowing, dance-like and musical actions, in vocalisation and in children's early drawings. This latter mode of representation is of especial power for the child because it is within the action of drawing (and please, please note that I am writing here of the child's *spontaneous*, self-initiated, self-guided drawing) that the child comes face-to-face with the awesome power of symbolic representation, that marks on a flat surface (whether these be physical pigment on a piece of paper, traces of light on a screen, or images on a liquid crystal display of a digital camera) are just that, yet simultaneously they refer to objects, events, ideas and relationships beyond the drawing surface.

Tragically, these profound beginnings of symbolic thought are still, in the main, discounted as 'scribbling.' Misguided attempts to 'improve' children's drawing and 'observational' skills, sometimes enlisting the support of so-called 'art specialists' make matters worse, cutting across, as they do, a crucial sequence of semantic and organisational principles spontaneously emerging on the drawing surface.

Sometimes my students ask me to recommend a good book on children's 'art'. I tell them to read the one you have started to read now, Carruthers's and Worthing-

ton's *Children's Mathematics*. The concept of 'children's art', with its inevitable train of consequences of 'art lessons' and 'art-specialists' in the early years, is at best, a mixed blessing. Definitional problems about the nature of visual representation have obscured the real meaning and significance of children's 2 dimensional visual structure (along with their interrelated investigations into 3 and 4D structures – the fourth dimension being the dimension of time). Many of the curriculum initiatives which bring dance, music and art 'expertise' into nursery are about as appropriate to children's development, and about as interesting to children, as mortgage agreements. Such initiatives merely add to the damage wrought upon children's emergent symbolisation.

Children's earliest drawing is generated spontaneously and is interrelated with many of their other modes of expression and representation. Although self-initiated and driven along by the child, it requires adult companions who are able to identify the operant modes of representation employed by the child. Such adults are therefore in a better position to supply intellectual and emotional support for the development of semiotic thought. Carruthers and Worthington not only identify the mathematical aspects of children's early modes of expression and representation, including drawing, they also show the teacher how these modes of representation may be best supported.

A careful reading of this fascinating book is quite simply the best way of understanding the growth of mathematical thought in infancy and how adult companions might nourish and support its development.

Dr. John Matthews

Professor

Visual & Performing Arts

Centre for Research in Pedagogy & Practice,

National Institute of Education,

Nanyang Technological University

Foreword

Chris Athey

This is a very important book not least because of its range. The authors have gathered evidence from children over a 15-year period. They analysed almost 700 samples of children's graphics showing how powerful patterns of cognition (schemas) in the early years of development gradually evolve into recognisable forms of writing and mathematics. Their aim, and unique achievement, has been to chart the progress of children's thinking through their mark-making from age 3 to 8. They have bridged the gap between Early Years and primary education.

When seen across such an age range, the children's explanations of the meanings of their own marks represent an exciting intellectual journey through childhood which will provide new insights for parents and professionals into the developing relationship between language and thought. The representations show a gradual emergence of more complex relationships between mathematical language and mathematical thought.

Evolving co-ordinations are vividly illustrated by children's own graphics and speech representations. In each case specific and appropriate references from the literature are given. These aid comprehension of complex material. The references are extensive and illuminative and specific page numbers are given at the end of quotations. This scholarly practice will be much appreciated by readers who may wish to pursue sub-themes in the book of which there are many: variations in pedagogy in different countries, working with parents and creating a mathematically stimulating environment are just a few.

The authors are vigorously in favour of school procedures which encourage children to be more participatory, and have greater autonomy, in their own learning. Many useful references are given in support of this constructivist pedagogical position.

One aspect of the enquiry shows that the majority of teachers still rely on mathematics worksheets where subject matter is neatly divided into discrete steps. Some of the children's cognitive confusions arising from these tasks are discussed. These confusions have to be seen against the clear conceptual understandings of children discussing their own invented symbolic systems.

There is nothing sentimental about the child-centred orientation of the attitude

held and evidence gathered by these two authors. They are tough teachers making a case for improving children's thinking, and mathematical thinking in particular. Their central thesis is that the gap in children's mathematical understanding is bridged through supporting the development of children's own mathematical graphics. At present there is a wide, conceptually dangerous gap.

Teachers, hopefully working with parents, can develop their own knowledge of early spontaneous patterns of thought in young children. Where adults learn the language and thought of young children they become better translators for the children into the language and thought of more formal mathematics. Adults are assisted by the children themselves who want to embrace more formal aspects of mathematics just as they wish to acquire more advanced strategies and skills in other areas of the curriculum. In translating between their informal and formal mathematical graphics children can exploit both. They will move with ease between their spontaneous ways of working things out, and their more newly acquired, more formal concepts. This is not a one-way movement: children move in an infinite loop as their translation supports them in becoming bi-numerate. Confidence will be maintained as competence increases.

The book is interestingly written and will strengthen professional knowledge on the development of meaning in children aged from 3 to 8.

Chris Athey
22 March 2006

Preface

In England over the last five years there has been some important government documentation. This has opened up tight and less flexible initiatives such as the National Literacy (DfEE, 1998) and Numeracy (DfEE, 1999) frameworks. In 2000 the introduction of the *Curriculum Guidance for the Foundation Stage* (QCA, 2000) clearly opened the way for more child-centred approaches and highlighted a play based curriculum. More importantly there was a move away from a subject based curriculum to the recognition that each subject was intertwined and therefore interdependent on the other. This holistic approach was further emphasised in the document *Excellence and Enjoyment: a Strategy for Primary Schools* (DfES, 2004c) in which it was stated that the numeracy and literacy frameworks were not statutory and that teachers should work flexibly within a broader curriculum. Schools were asked to ‘take control of the curriculum, and be innovative’ (p. 16). At the same time the national assessment procedures for Key Stage One were gradually moving more to teacher assessment in evaluating children’s attainment, thus recognising the teacher’s professional expertise and the knowledge she had of the children in her class.

The *Foundation Stage Profile* was introduced in 2003 and sits well with the *Curriculum Guidance for the Foundation Stage*. This profile reports children’s attainment at the end of the Foundation Stage and is based on ongoing observations of children throughout the year, rather than the very narrow task testing procedures of the previous ‘base line assessment’. This kind of assessment not only helps teachers to know children’s achievements but also informs the learning process. The observation based profile is important for teachers to judge the outcomes and therefore the quality of play. Documenting what children say and do in play has highlighted for many, who may have needed convincing, that children are challenged to the maximum of their capabilities in play. This has helped people realise the cognitive potential of play and of a play based curriculum. However, moving towards a play based curriculum has meant that the downward pressure of a more formal curriculum in the Primary sector did not match the ethos and principles of the Foundation Stage. Transition from one key phase to the other has been reported as being detrimental to young children (NFER, 2005). To counter this negative effect the training

materials encompassed by *Continuing the Learning Journey* were produced for schools, (QCA, 2005). This has been welcomed by Early Years professionals as the materials emphasise continuing the play based approach in year one and planning from children's interests, as well as looking at the objectives needing to be taught.

Again from 2006 the documentation and guidelines are to be reviewed. There is a need to move further towards a more holistic approach to children's learning and teaching. The *Every Child Matters* agenda (DfES, 2004a) has been a catalyst for change and is underpinning the *Early Years Foundation Stage* (forthcoming) where *Birth to Three Matters* (DfES, 2002a) and the *Curriculum Guidance for the Foundation Stage* (QCA, 2000) are meshed. The numeracy and literacy frameworks are being reviewed with consideration to the *Early Years Foundation Stage* document. Many quality government supported materials have been produced, for example *Listening to Young Children* (Lancaster and Broadbent, 2003) and *Communicating Matters* (SureStart, 2005). All these documents should help put the child and their family back at the centre of the learning.

The rise of Children's Centres as an almost organic approach to education and care is an exciting initiative and will be both a breeding ground for new thinking and providing a new research base. Children's Centres play a key role in the implementation of the government's ten year strategy for childcare (DfES, 2004b). These centres are becoming internationally renowned as an up-to-date model of early education and care supported by a multi-disciplinary team. The 'British Infant School' model of the sixties and seventies had similar acclaim as a pioneer of new thinking with the influence of Piaget and a play centred curriculum. These new centres are the perfect context to open up teaching and learning with a strong emphasis on practitioner research.

Against this background this book will add to the revival of looking at young children more closely. The area of mathematics is still riddled with questions and some of the main ones that concern teachers are, 'How can I move children to understanding the abstract symbolism of mathematics? What is the development? When and how do you introduce standard symbols? What do children's own mathematical graphics look like?' We are at a time of giving teachers back their professionalism and allowing them to really observe young children and to support their own thinking and meaning making: this is the key to teaching and learning about children's mathematical mark making.

Since we wrote the first edition we have noted that settings for children under five are well on the way to creating the body of knowledge needed to support children's thinking in mathematics. However, as this Advanced Skills (reception) teacher explained:

I had long felt frustrated with some aspects of maths 'teaching' for reception children. There were exceptions of recording work, even though nearly all of the maths was practical in nature. You can take photographs of some activities, but that only records the doing, not the thinking ... I began to question why I had not considered maths (in the same way as emergent writing). Young children don't learn in convenient blocks, defined by subject areas (sorry Ofsted). They

learn from experience. They pull together bits of knowledge they have gained: they observe; they try things out; they learn from asking questions; they experiment; they work at their understanding until they make a connection with another bit of knowledge they have (Jacoby, 2005: p. 38).

Schools are not generally this far advanced and this is not only where the real challenge is but, paradoxically, this could be where the most benefits lie. If schools also develop understanding of children's own mathematical graphics, then the continuity of their mathematical thinking from pre-school to school will give children the ability to really understand and use the abstract symbolism of mathematics. If we do not nurture children's own thinking in schools then the work of pre-schools will not be realised and many children will still be confused with the standard algorithm.

The publication of the first edition of this book in 2003 re-opened the debate about 'emergent' mathematical approaches and interest in children's mathematical graphics has subsequently multiplied in England, the UK and internationally. Increasingly we meet and hear from teachers who are discovering for themselves the potential that mathematical graphics holds for children's understanding of 'written' mathematics and how this supports children's thinking and learning of mathematics at a deep level.

Learning and using abstract symbols and written calculations with understanding can be challenging for young children unless teaching approaches support this development. We are the first to have created a taxonomy of children's visual representations of their mathematical thinking from birth to eight years. By listening to teachers we have developed the taxonomy further since the publication of our first book and this can prove invaluable for teachers' understanding and assessment (see p. 131).

We hope that this book will encourage you to begin to make at least small changes in your approach to teaching 'written' mathematics, so that you too will marvel at the depth of young children's early mathematical thinking and understanding.

