Contents

Foreword by Christina Tondevold xix
Preface xxi
Acknowledgments xxvi
About the Authors xxviii

1 WHAT DOES FLUENCY REALLY MEAN, AND WHY DOES IT MATTER? 1

WHAT IS FLUENCY IN MATHEMATICS? 1
FLUENCY ACTIONS 4
CHECKS FOR REASONABLENESS 9
FLUENCY FOR EACH AND EVERY STUDENT 15
MATHEMATICS IDENTITY AND AGENCY THROUGH FLUENCY 16
PRODUCTIVE BELIEFS ABOUT ACCESS AND EQUITY FOR FLUENCY 18
EFFECTIVE TEACHING PRACTICES FOR FLUENCY INSTRUCTION 19
FIGURING OUT FLUENCY: SETTING CLEAR GOALS 20

2 FLUENCY FALLACIES AND RELATED TRUTHS 23

LANGUAGE FALLACIES 23
FLUENCY FALLACY #1: *FLUENCY IS ABOUT BASIC FACTS.* 24
FLUENCY FALLACY #2: *MASTERY, FLUENCY, AND AUTOMATICITY ARE THE SAME THING.* 25
FLUENCY FALLACY #3: *REPRESENTATIONS ARE STRATEGIES.* 26
STANDARD ALGORITHMS 28
FLUENCY FALLACY #4: *STRATEGIES AND ALGORITHMS ARE THE SAME THING.* 28
FLUENCY FALLACY #5: *ONCE LEARNED, THE STANDARD ALGORITHM IS THE BEST CHOICE.* 29

ACCESS AND EQUITY 31

FLUENCY FALLACY #6: *STANDARD ALGORITHMS ARE STANDARD EVERYWHERE.* 31

FLUENCY FALLACY #7: *SOME STUDENTS ARE BETTER OFF WITH KNOWING JUST ONE WAY.* 33

TEACHING AND ASSESSING 35

FLUENCY FALLACY #8: *PROCEDURAL FLUENCY PRACTICE IS A LOW-LEVEL COGNITIVE EXPERIENCE.* 36

FLUENCY FALLACY #9: *FIRST LEARN CONCEPTS, THEN LEARN PROCEDURES.* 38

FLUENCY FALLACY #10: *THERE IS A SET LIST OF STRATEGIES FOR ANY GIVEN TOPIC.* 40

FLUENCY FALLACY #11: *SKILLS TESTS ASSESS FLUENCY.* 41

FLUENCY FALLACY #12: *CONCEPTUAL UNDERSTANDING IS MORE IMPORTANT THAN LEARNING SKILLS.* 42

FIGURING OUT FLUENCY: REPLACING FALLACIES WITH TRUTHS 42

3 GOOD (AND NECESSARY) BEGINNINGS FOR FLUENCY 47

CONCEPTUAL UNDERSTANDING 48

PROPERTIES AND UTILITIES FOR STRATEGIC COMPETENCE 52

DISTANCE FROM A 10 52

DECOMPOSING NUMBERS FLEXIBLY 53

PART–PART–WHOLE 55

SKIP COUNTING 56

DISTRIBUTIVE PROPERTY 57

GOOD BEGINNINGS: BASIC FACT FLUENCY 59

#1 A STRATEGY-BASED APPROACH LEADS TO "BETTER"-LEARNED FACTS 59

#2 FACTS STRATEGIES ARE NEEDED BEYOND BASIC FACTS 64

#3 LEARNING BASIC FACT STRATEGIES DEVELOPS MATHEMATICAL AGENCY 66

COMPUTATIONAL ESTIMATION 67

COMPUTATIONAL ESTIMATION STRATEGIES 68

TIPS FOR ADVANCING COMPUTATIONAL ESTIMATION 72
THE EFFECT OF OPERATING ON NUMBERS 73
FIGURING OUT FLUENCY: GOOD BEGINNINGS 74

4 SEVEN SIGNIFICANT STRATEGIES FOR DEVELOPING FLUENCY 76

DEVELOPING STRATEGIC COMPETENCE 77
1. COUNT ON/COUNT BACK (ADDITION AND SUBTRACTION) 78
WHAT IT IS 78
VISUALS TO SUPPORT 78
SITUATIONS THAT CONNECT 79
2. MAKE TENS (ADDITION) 80
WHAT IT IS 80
VISUALS TO SUPPORT 81
SITUATIONS THAT CONNECT 83
3. USE PARTIALS (ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION) 84
WHAT IT IS 84
VISUALS TO SUPPORT 88
SITUATIONS THAT CONNECT 89
4. BREAK APART TO MULTIPLY (MULTIPLICATION) 89
WHAT IT IS 89
VISUALS TO SUPPORT 91
SITUATIONS THAT CONNECT 93
5. HALVE AND DOUBLE (MULTIPLICATION) 93
WHAT IT IS 93
VISUALS TO SUPPORT 94
SITUATIONS THAT CONNECT 95
6. COMPENSATION (ADDITION, SUBTRACTION, AND MULTIPLICATION) 96
WHAT IT IS 96
VISUALS TO SUPPORT 98

7. USE AN INVERSE RELATIONSHIP (SUBTRACTION AND DIVISION) 101
WHAT IT IS 101
VISUALS TO SUPPORT AND SITUATIONS THAT CONNECT 101
EXPLICIT STRATEGY INSTRUCTION 102
FIGURING OUT FLUENCY: SEVEN SIGNIFICANT REASONING STRATEGIES 104

5 AUTOMATICITY BEYOND THE BASIC FACTS 107

AUTOMATICITY 108
LEARNING PROGRESSION FOR AUTOMATICITY WITH BASIC FACTS 108
AUTOMATICITY AND FLUENCY 109
TIPS FOR DEVELOPING PROCEDURAL AUTOMATICITIES 111
PROCEDURAL AUTOMATICITIES 112
BREAKING APART ALL NUMBERS THROUGH 10 113
BASE 10 COMBINATIONS 115
USING 25s 116
USING 15s AND 30s 118
DOUBLING 119
HALVING 120
FRACTION EQUIVALENTS WITHIN FRACTION FAMILIES 123
CONVERSIONS BETWEEN COMMON DECIMALS AND FRACTIONS 125
FIGURING OUT FLUENCY: PROCEDURAL AUTOMATICITIES 127

6 FLUENCY PRACTICE IS NOT A WORKSHEET 130

CHARACTERISTICS OF QUALITY FLUENCY PRACTICE 131
QUALITY PRACTICE IS FOCUSED 131
QUALITY PRACTICE IS VARIED 133
QUALITY PRACTICE IS PROCESSED 134
QUALITY PRACTICE IS CONNECTED 136
FREQUENTLY ASKED QUESTIONS ABOUT FLUENCY PRACTICE 137

TYPES OF FLUENCY PRACTICE 139
FLUENCY ROUTINES 139
WORKED EXAMPLES 144
GAMES 147
CENTERS 150
INDEPENDENT PRACTICE 152
FIGURING OUT FLUENCY: QUALITY PRACTICE 152

7 BEYOND ACCURACY: WHAT WE NEED TO ASSESS AND THE TOOLS TO DO IT 154

ASSESSING FLUENCY 154
PLANNING FOR AND ASSESSING STRATEGIES AND AUTOMATICITIES 155
ASSESSING FLUENCY COMPONENTS AND ACTIONS 159
GRADING FLUENCY 163
ASSESSMENT OPTIONS BEYOND QUIZZES AND TESTS 164
OBSERVATIONS 165
STUDENT INTERVIEWS 167
JOURNAL PROMPTS 169
SELF-ASSESSMENTS 172
FIGURING OUT FLUENCY: ASSESSING FLUENCY 174

8 ENGAGING FAMILIES IN FLUENCY 176

FREQUENTLY ASKED QUESTIONS ABOUT FLUENCY INSTRUCTION 177
FAQ #1: *WHY DOES FLUENCY MATTER FOR MY CHILD?* 178
FAQ #2: *WHEN WILL THEY EVER NEED ALL OF THESE STRATEGIES?* 178
FAQ #3: *HOW WILL THIS HELP MY CHILD DO WELL ON THE TESTS THEY HAVE TO TAKE?* 178
FAQ #4: *WHY IS THIS MATH SO DIFFERENT FROM WHAT I LEARNED?* 179
FAQ #5: *WHY ARE BASIC FACTS AND THE PROCEDURES WE LEARNED NOT TAUGHT ANY LONGER?* 179
FAQ #6: *WHY DOESN'T SPEED MATTER ANYMORE?* 180
FAQ #7: *MATH IS RIGHT OR WRONG. SO WHY DOESN'T THE RIGHT ANSWER COUNT ANYMORE?* 180

FAQ #8: *WHY DO THEY HAVE TO SOLVE A PROBLEM IN SO MANY DIFFERENT WAYS?* 180
FAQ #9: *DON'T ALL OF THESE STRATEGIES JUST CONFUSE CHILDREN?* 183
FAQ #10: *WHY DON'T CHILDREN PRACTICE BASIC FACTS AND ALGORITHMS A LOT (LIKE WE DID)?* 183
TIPS FOR RESPONDING TO FAQs 183
TAKING ACTION TO INFORM AND ENGAGE FAMILIES 184
BACK-TO-SCHOOL AND ORIENTATION EVENTS 184
NEWSLETTERS, CLASSROOM UPDATES, AND WEBSITES 186
FLUENCY BROCHURES 187
FAMILY FLUENCY EVENTS 188
VIDEO SHORTS 189
CLASSROOM VISITS 190
FIGURING OUT FLUENCY: ENGAGING FAMILIES 190

9 FLUENCY IS THE FOCUS: PLANNING, AGREEMENT, AND ACTION 192

PLANNING FOR FLUENCY 192
UNIT PLANNING FOR FLUENCY 194
DAILY PLANNING FOR FLUENCY 196
COLLABORATIVE AGREEMENTS: ADAPTING, ADOPTING, AND ELIMINATING 199
FIGURING OUT FLUENCY: TAKING ACTION 209

Appendix A: Activity List 210
Appendix B: Strategy and Automaticity Reference Page 213
References 214
Index 222

online resources

Visit the companion website at **resources.corwin.com/figuringoutfluency** for downloadable resources.