CHAPTER 1

Understanding Subtraction through Enhanced Communication

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Communicating the Theme

> Sam – a young and enthusiastic trainee teacher – asked the class, 'What is the difference between 7 and 6?'. Jo's hand shot up and he immediately responded, 'Well seven is all straight lines and sixes are all curly'.

Teaching subtraction is harder than teaching addition for several reasons. One – as graphically illustrated by Jo's response – is the use of words which have different meanings in different contexts: 'difference', 'take away', 'the same as', to name but a few. Another reason why subtraction can be difficult to teach is that people have a tendency to make some kinds of comparisons and not others. For example, a child will quite happily say, 'I am taller than my brother' or 'I have more pencils than Sandy'. But, how often have your heard comments such as, 'It's not fair, I've got 8 less than him!', or 'It's wonderful those cakes cost much more than those ones'. In this chapter I will explain more about subtraction and explore why some children find it hard to understand. I will then provide a selection of activities which encourage a range of different communication strategies which have been used successfully with children and adults both in the UK and the other COSIMA countries.

Communicating the Concepts

We all use subtraction frequently. Most of us probably use it everyday…

- when you buy five cream cakes, eat one and wonder whether you will have enough of them left for your visitors;
- when you compare your results on a test with a colleague;
- when you work out whether you have enough money to buy something.

My observations suggest, however, that surprisingly few people understand the range of basic concepts involved. If you had to describe subtraction what would you say? Would it be along the lines of 'taking away' or 'finding the difference between two values'? Would you add anything else or do you think that covers it? When I asked 168 primary teacher training students at the beginning of their mathematics course, 75% defined subtraction as 'taking away', 8% said that it was 'the difference between two numbers' with a further 9% considering it to be both. Of the remaining 8%, only one gave a slightly more complete explanation saying that, 'Subtraction can mean various things such as 'take away' or 'less than''.

In the past, subtraction was typically taught as 'taking away': it was easy to demonstrate and children could quickly relate it to their own experiences of 'taking away' – or eating – biscuits; 'taking away' – or losing – toys and so on. More formally this type of subtraction is called *partitioning*: you start with a certain number of items, some are removed in some way (e.g. eaten, spent, stolen) and the question is, 'How many are left?'. This is illustrated by the cream cake example above and in Figure 1.1a. The partitioning method of subtraction is introduced to four- and five-year-old children in the UK in the first year of formal schooling.

Typically pictorial representations of partitioning involve two groups of items distinctly separated as in Figure 1.1a but it is important to provide children with a range of illustrations – such as in Figures 1.1b and 1.1c – to demonstrate that the same principle applies whatever the arrangement of the items presented. Ideally teachers can demonstrate the processes involved in subtraction by physically undertaking a variety of activities such as eating a cherry from a bag of ten; breaking a biscuit from a newly opened packet and asking how many whole ones are left and so on.

Recently UK teachers have been encouraged to teach five- and six-year-olds 'difference between' – also known as the *comparison structure*. This is the structure you would use when comparing marks in a test as described above. Children typically use this structure when comparing their ages or who has more biscuits (see Figure 1.2a). Questions relating to this structure are generally of the type:

- 'What is the difference between… ?'
- 'How many more in… ?'
- 'How many fewer in… ?'

Interestingly, children tend to use 'more' rather than 'fewer' in their conversations but it is important that teachers give them experience of both, emphasising, for example, that in Figure 1.2a Sam has three fewer biscuits than Jo, as represented by $5 - 2 = 3$, and Jo has three more sweets than Sam, also represented by $5 - 2 = 3$.

Again, I would suggest that learners are provided with a range of representations and examples so that they develop a deeper understanding of the comparison structure. The examples given in Figure 1.2b are good starting points for a discussion. With older children comparisons can also be made in a range of settings where pictures provide less tangible support: such as comparing the speeds of cars or the volumes of irregular objects.

Figure 1.1a *An example of the partitioning method of subtraction*

Figure 1.1b *An alternative example of partitioning*

Figure 1.1c *Another example of partitioning*

Figure 1.2a *An example of the comparison model of subtraction*

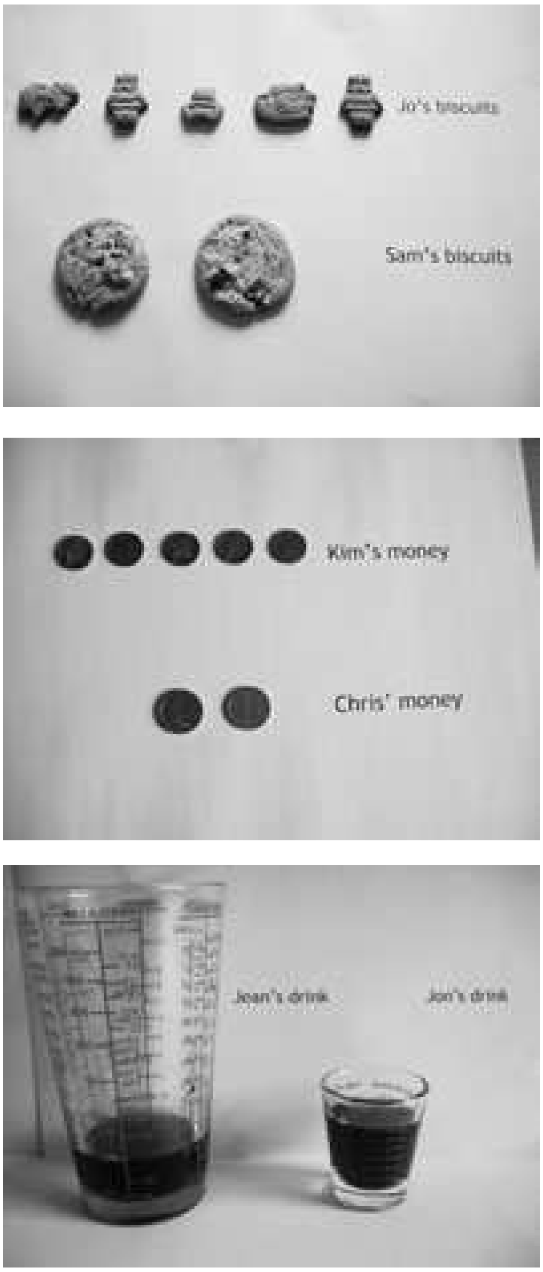


Figure 1.2b *Other examples of the comparison model of subtraction*

In Year 2 (six- and seven-year-olds) UK children are often introduced to a third type of subtraction: *the inverse of addition structure*. This is illustrated by the earlier example when you wanted to know whether you had sufficient money to buy something. This is usually rather more difficult than the partitioning and comparison structures for children as it typically involves questions such as, 'How much more do I need?' or 'What must be added to 3 pence to make 10 pence?'. As discussed further below, in such cases children frequently latch onto words such as 'more' and 'added' and therefore they frequently add rather than subtract. A further danger looms if you wish to use a practical demonstration. For example, if you produce both 3 pence and 10 pence (as you might if it were a comparison), you might be given the answer 13 pence when what you really wanted to know was: if you had 3 pence how much more money would you need to buy a sweet costing 10 pence.

There are two other structures primary children need to become familiar with if they are to have a good grounding in subtraction before they proceed to more advanced mathematics. The first is the *complement of a set – or union – structure* which may, at first sight, appear rather awkward but which, in reality, is used quite frequently in everyday life. Here the word 'not' – or something similar – is typically used. For example:

- I have made 8 sandwiches, 5 already have butter on them, how many do not?
- There are 25 children on the class register but only 22 children came in today, how many are not here?

The other type of subtraction involves the *reduction structure* and this is probably most commonly encountered during the sales when items are reduced: a coat costs £100 in December but is reduced to £75 in the January sales. In many ways the reduction structure is similar to that of partitioning but there is one notable difference. When taking away – or partitioning – you are working with problems which may be represented with real objects but, when reducing, this need not necessarily be the case. For example, yesterday the temperature was 6°C but overnight it dropped by 10°C; what is the temperature now? A number line can be a good way to illustrate such problems, as shown in Figure 1.3. As will be discussed further below, although they are similar, experience with the both reduction and partitioning structures is crucial if children are to have a sound understanding on which to build when they encounter more advanced mathematics and, in particular, algebra.

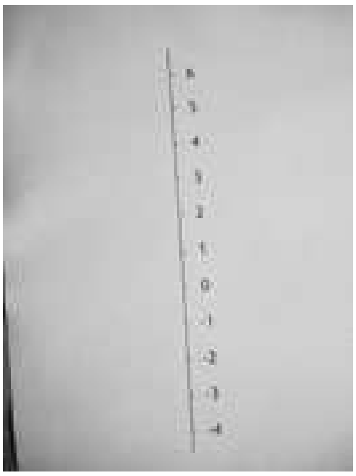


Figure 1.3 *Example of how a number line might be used when working with the reduction structure*

Prior to recent reviews of primary mathematics teaching in the UK, teachers tended to focus on the products of children's work making it difficult to assess their understanding. Indeed, in my work with Charles Desforges in the 1980s, we discovered that in almost one in four cases, children were not solving calculations in the way their teachers had anticipated: some were getting the correct answers without doing any mathematics whatsoever! For example, some individuals copied someone else's work while others looked back at earlier examples of their own work that had already been marked by the teacher. More often that not, children's ploys went undetected by their teachers. Also, as discussed above, sometimes children's responses were constructed using different information to that predicted by the teacher. For example, Mrs T asked her class of five-year-olds, 'If there were 5 apples in the fruit bowl and you ate 2 of them, how many would be left?'. Mia replied '4'. When Mrs T asked Mia to explain she said, 'I'm never allowed to take 2 apples so I only ate one and so there were 4 left'.

Now, UK teachers are expected to invite their pupils to explain how they arrive at their solutions, encouraging the children to focus on their thinking processes and understanding. Some do this well; others less so. Sometimes the latter teachers lack confidence in their own mathematical abilities. Sometimes they prefer to continue using the more traditional methods in which they were trained. Sometimes they think they are providing opportunities for children to share their ideas and explain their different strategies when, in fact, they may be encouraging pupils to use – and discuss – a specific method. Take, for example, the case of Mrs R[1] and her class of five-year-olds.

Mrs R: Who can tell me the answer to 20 subtract 11?

Jude: 9

Mrs R: Good! Now tell us how you did it?

Jude: Well I counted back from 20 to 11 and got to 9.

Mrs R: Mmm. Did anybody do it any other way?

Sally: Well I put 11 in my head and counted up to 20 using my fingers and I ended up with 9 fingers.

Mrs R: Mmm. Any other suggestions on how you might subtract 11 from 20? Remember what we did yesterday.

Molly: Well I know that 20 minus 10 is 10 and 11 is one more than 10 so the answer must be 9.

Mrs R: Great. Well done Molly. Now everyone, remember those number facts you learnt last night when you are doing your maths today.

Mrs R's actions were understandable if her principle aim was to teach her class an efficient way to subtract using known number facts; however, in guiding her class towards her preferred method she may be discouraging them from thinking through – and retaining – potentially successful strategies for themselves. Thinking about how you learnt to use a computer may illustrate the point more vividly. Taking my own case, my most effective learning has been when I have had to struggle through trial and error rather than through having a computer genius show me which buttons to press. Returning to the classroom, Mrs R's technique may not work so well if you have a poor memory coupled with a lack of understanding of subtraction. If, however, children are exposed to a range of strategies – including their own – they can begin to develop a greater appreciation of how numbers can be manipulated and more of an understanding of how they might tackle subtraction problems in a manner which suits their current knowledge and expertise. (As an aside, the learning of times-tables seems to go in and out of fashion: it is fine if you know that 90mm x 70mm is 6300mm when you are making a rectangular cake but what can you do if you cannot remember that 9 x 7 = 63 and you have no calculator to hand? Having the facility to both estimate and work from first principles means that you are likely to be much more successful when you attempt to work out whether the size of the cake you ordered is better suited to an individual or a large party.)

One of the underlying principles of this book is that children develop authentic mathematical understanding if they construct meaning from their own mathematical efforts while, at the same time, observing and reflecting on others' ideas and strategies. Thus, it may be that they do not tackle subtraction problems in the most efficient way to start with. Rather they complete their work in a way that has meaning for them and through a range of processes – discussions with teachers, other children, games, everyday life, observations and so on – they construct and refine the processes they use until they are both effective (i.e. they get the calculations correct through the application of their understanding) and efficient (i.e. they can solve problems quickly). Without such an evolution in their mathematical thinking we would argue that the chances of children making errors are much higher and they may well have inadequate foundations on which to build.

To summarise: there are several different subtraction structures and, in order to develop a sound understanding of the concepts involved, it is important to use and discuss a variety of them.

In the remainder of this chapter I will provide a range of strategies that we have invited students and teachers to use both in UK and elsewhere. The focus in all cases is to introduce primary children and both future and experienced teachers to a wealth of mathematical opportunities in a child-centred manner.

Communicating Experiences with Pre-service Teachers

As discussed above, few trainee teachers have a sound understanding of subtraction. Obviously the various structures can be described to them but, as with children, to gain a real appreciation of subtraction, they need to be exposed to a range of experiences and construct their own knowledge and understanding of the concepts involved. In this section I will present some tried and tested strategies we have used with student teachers both in the UK and elsewhere.

Being able to visualise situations can prove very helpful when trying to decide how to respond to a given problem. For example reflect on what goes on in your mind when I say, 'I had 6 eggs, I used 2 to make a cake, how many did I have left for breakfast?'. As a warm-up activity you might ask students to match the pictures and problems given in Figure 1.4 and discuss their responses with a partner. Assuming that this activity presents few – if any – difficulties, you could ask the students to match the problems given in Figure 1.5 to the calculations presented. You could then encourage them to sketch pictures of the problems. Such a task very much mirrors what students might be required to do with young children and it also provides a good indication of their understanding. Discussion with a student partner should be encouraged as it presents an ideal opportunity to talk mathematically and articulate thinking. Hopefully, through conversation, the students will begin to understand that, although pictures can prove extremely useful, it is not always easy to represent mathematical problems pictorially in an accurate and helpful way. For example the following is misleading:

6	take away	2	equals ?
******		**	

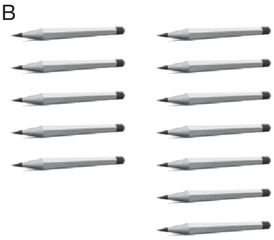
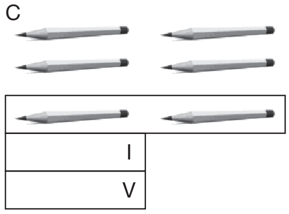
Problems	Pictorial representations
1 This morning I had 6 pencils. Tom took 2 away. How many do I have now?	A 
2 I have four pencils. Mary has 6 pencils. How many fewer pencils do I have than Mary?	B 
3 I need to have 6 pencils for my lesson tomorrow. I only have 4. How many more do I need?	C 

Figure 1.4 *Sample of matching tasks for trainee teachers*

Problem	Calculation
1 John has 8 cars. 3 are large. If all the others are small, how many are small?	A ? + 3 = 8
2 Simon has 8 pencils. He loses 3. How many does he have left?	B 8 = 3 + ?
3 Bianca has some marbles. She bought 3 more. Now she has 8. How many did she have to start with?	C 8 – 3 = ?

Figure 1.5 *A sample of problems and the corresponding calculations to match*

As I will discuss further below, I am against young children formally committing their mathematics to paper before they have a very solid grounding in the concepts involved. This notwithstanding, it is important that future primary teachers can convert mathematical problems into equations and vice versa. Thus, for example, we divide trainees into two groups. The first are given the problems shown in Figure 1.6 and the second the number sentences. The tasks are either to produce appropriate equations or create suitable problems to match the problems and then compare and contrast the results. Such exercises can prove to be useful catalysts for discussion about the equals sign (=) and the difficulties some children encounter when faced with examples such as 10 = 6 + ? (see also below). It is also worth noting that discrete (i.e. involving countable objects) and continuous (e.g. ages and weights) examples are used in Figure 1.6. It is important that trainee teachers and their future pupils become familiar with both. In passing, if you were using a calculator, all of the problems would be solved by visualising them as 10 – 6 =?

Problem	Number sentences
1 Sarah wants 10 cars. She already has 6. How many more does she need?	6 + ? = 10
2 (a) Jenny is 10 years old. David is 6. What is the difference in their ages? (b) Fred has a statue which weighed 10 kilograms. The head dropped off. It weighed 6 kilograms. How much does the statue weigh now?	10 – 6 = ?
3 At 9am Bryony had a bucket of water. She added 6 litres. Now there are 10 litres in the bucket. How much water did she have in the bucket at 9am?	? + 6 = 10
4 Miss Brown would like 10 of her students to understand subtraction. She knows that 6 of them already have a good understanding. How many will she need to teach?	6 + ? = 10

Figure 1.6 *Example discussion exercise in creating equations and problems*

As part of their training it is also important that student teachers become familiar with resources commonly used in primary mathematics classrooms. These tend to go in and out of fashion and include equipment such as number lines, multilink and cuisinaire rods. Trainees' familiarity with these resources may depend on their age and where they attended primary school. For example, until relatively recently the number line was frequently used in mainland Europe but was rarely used in UK primary classrooms. Extending the activities associated with Figure 1.6 to include consideration of appropriate resources is a very informative exercise and, again, can reveal who has a good understanding of subtraction problems and the ways in which they might best be represented. The DVD (Item 1.1) includes examples of how student teachers in Germany used Duplo to illustrate various subtraction structures involving 8, 5 and 3.

A key part of a student teacher's preparation is their time spent in school. We very much encourage our students to observe experienced teachers in action, note their techniques and discuss the thinking behind sessions. Unfortunately, as illustrated above with Mrs R, teachers do not always operate in a way we would recommend and thus it is important that students discuss their observations on their return to college and consider why teachers sometimes behave in unexpected ways.

Working in schools provides an excellent opportunity for students to see young children engaged in mathematical activities. For many years we have asked trainees to note down any mathematical errors pupils make. A sample of these is presented in Figure 1.7a. As part of the task the students are encouraged to talk to their pupils and pose questions which will elicit the children's thought processes. This exercise seems to be another effective way to assess trainees' understanding but it also encourages them to think beyond the children's responses and begin to consider how a learner might be thinking and why they might be thinking like that. This, in turn, helps trainees to reflect on their own practice and the opportunities they might provide for their pupils to reduce the likelihood of errors. Thus, if a child who wrote $101 - 9 = 2$ had had more experience of associating quantities of items to their numerical representation, they might have realised that their response of 2 was inappropriate. The error was almost certainly due to an insufficient understanding of place value rather than subtraction but, as is often the case, misconceptions may come to light unexpectedly. As an aside, all too rarely in my view do children see – let alone touch – more than about 20 objects in a classroom activity so it is not entirely surprising that they have little understanding of place value and the quantities large numbers represent.

Task	Child's response
Zubin had 7 sweets. Jack had 10. How many more sweets did Jack have than Zubin?	7 + 10 = 17
Sally scored 6 goals and Natasha scored 14. What was the difference between the scores?	6 – 14 = 0
52 – 47 =	52 – 47 = 15
101 – 9 =	101 – 9 = 2
3 – 2 = *** **	5
Using a number line calculate: 8 – 5 =	4

Figure 1.7a *Examples of children's mathematical errors*

Having considered Figure 1.7a, how would you explain the errors? Take a minute to reflect on your responses and then you might like to compare them with those of the students as presented in Figure 1.7b.

Child's response	Student teachers' explanation
7 + 10 = 17	Zoe had difficulty reading the problem but she recognised the word 'more' and so she added 7 and 10.
6 – 14 = 0	Ben wrote down the numbers in the order they were given and explained that the answer must be 0 as he had been presented with an impossible situation.
52 – 47 = 15	Mia has an insufficient understanding of place value and, for example, thinks of 52 as 5 and 2. The problem is compounded by the fact that she believes that you always take the smaller number from the larger. Hence she did 5 – 4 and then 7 – 2.
101 – 9 = 2	John does not appreciate that 0 acts as a place holder and has a significant part to play, thus 101 is not 11.
5	The stars underneath the figures were intended to remind Sam of the values associated with 3 and 2. Sam was delighted when she explained that there was no need to look at the numbers as all she had to do was count the stars.
4	I observed Kai using the number line but incorrectly i.e. he started at 8 and counted all the numbers back to, and including, 4.

Figure 1.7b *Student teachers' explanation for the errors shown in Figure 1.7a*

To summarise: successful trainee teachers need to acquire a sound understanding of subtraction and all its many guises. They need to become comfortable discussing – with colleagues and children – the teaching and learning issues surrounding it and adept at demonstrating a range of ways to illustrate the various subtraction structures.

Communicating Activities for the Classroom

As mentioned above, young children have experience of subtraction in their everyday lives. As a teacher it is important that you capitalise on these experiences so that your pupils can build on their developing understanding of the concept. There are several ways to do this many of which will be familiar to experienced teachers. For example you might invite the parents in for an evening of primary mathematics. Recently a local school did this and they were overwhelmed by the numbers who attended and their responses. Their success was in part due to excellent publicity and in part to the exciting activities the parents were invited to try. I suspect quite a few were surprised by how much they learnt and how little they had previously understood! Joint home/school projects can also prove very successful, although care needs to be taken that parents do not engage in too much ‘teaching’ as this can result in what Constance Kamii has described as ‘mindless techniques that serve only as tricks’ (1985: 103). Activities which can encourage home/school teamwork might include baking, model making and games (see below) all of which can include elements of subtraction and opportunities for much discussion. Talking to a colleague – Ralph – over coffee, he immediately volunteered that playing darts was an excellent way to learn subtraction: motivation is high and the pressures sometimes associated with the mathematics classroom are low. Having a number line and a 100-square close by can prove handy aids initially although these may raise some eyebrows if your darts playing is in a pub!

Everyday activities in school are a useful way to demonstrate the relevance of subtraction in real life situations. This need not be either time consuming or a major event – although it can be should you wish – but rather it is an opportunity for the children to see you use mathematics and to observe you adopting a range of strategies to solve problems. As a brief aside: from time to time I ask primary pupils why we need to learn about numbers and whether they have ever seen adults use them. A response which still sticks out in my mind – albeit more than 20 years after it was given – came from Ben who explained that his Mum had last used numbers ‘… about nine months ago … no … a year ago, when we moved house, Mummy had to count pegs to put the curtains up with’ (Desforges and Cockburn, 1987: 100). Amusing perhaps but also somewhat disturbing as this is one of the few responses I have ever encountered where a child has been able to describe seeing an adult using numbers. To return to the classroom: you could ask your pupils why they use numbers and, indeed, whether they have observed adults doing so. You might ask them – as Jackie did – for their definitions of subtraction. Interestingly her class of seven- and eight-year-olds gave more impressive answers than the student teachers mentioned above! More specifically the children described subtraction as:

Taking away

Opposite to adding

You take a number from a higher number

It’s counting back

Dividing is repetition subtraction

Minus

Find the difference

You might find it revealing to ask the class to write number stories to match each of the above words and phrases.

There are everyday opportunities to use subtraction when taking the register (e.g. you can practice partitioning when you ask: ‘There are 25 names on the register. 5 children are absent. How many children are here today?’[2]); when handing out resources (e.g. you could focus on the complement of a set structure when you pose the following question: ‘I have 12 crayons. I am going to give 6 of them to Bob. How many will be left for me?’[3]) and when setting up teams (e.g. the comparison structure may be illustrated by: ‘There are 5 children in group A but only 3 in group B. How many more do I need in group B to make 5?’) and so on. Sometimes you might want to model the technique you use to solve the problem – for example by drawing a quick number line and counting back the number of children who are absent. Sometimes you might decide to invite the children to explain their strategies for reaching a solution and sometimes you might not say anything at all because time is short which, in effect, allows you to model another way in which calculations may be done, that is quickly and quietly in one’s head. Songs and poems are another mathematically low stress way to practice subtraction. Sometimes to add a bit of variety and fun you, as the teacher, can make a deliberate mistake. Our observations suggest that children very much enjoy this and are only too willing to explain in detail what you have done wrong and why. Although I suggest usually doing these activities in a low key way I would advise that, from time to time, you remind the children that they are doing subtraction. Jackie, for example, discovered that her class were amazed to learn that they had actually been subtracting successfully when some of them found 'mathematics' something difficult and to be avoided.

More formal opportunities for communicating subtraction strategies can occur in oral mental starters at the beginning of mathematics sessions. Here it is a question of sharing ideas and strategies rather than a teacher imposing his or her view of subtraction and how it might be done. The follwoing extract shows David working with a class of eight-year-olds on a range of subtraction structures at the start of a mathematics session.

T: *Now the questions are not difficult. What I am interested in is how you explain what you have done. Here we go. 'Peter has 26 pieces of fruit. 9 are apples. How many pieces are there of other types of fruit?'. [Children write their responses on individual boards and hold them up for the teacher to see.] Okay we've all got the same answer. Tim, what calculation did you actually do?*

Tim: *I minused 10 and added on one.*

T: *So you minused 10 from 26 and added on one. Did anybody do it a different way? Sam?*

Sam: *Well I know that 6 is just 3 less than 9 and take away 3 from 20.*

T: *Explain that again for me.*

Sam: *6 is 3 less than 9.*

T: *Why is that important to you then? Why is it important that 6 is 3 less than 9?*

Sam: *'Cos if I take away 3 from 20 ...*

T: *From 26? So you counted back 6 then you counted back another 3?*

Sam: *Yes.*

T: *Here is the second one: 'I made 35 cakes and I ate 7 of them, how many are left?'. Caitlin how did you do it?*

Caitlin: *I know that 7 was 2 more than 5 so I took 5 from 35 and I took 2 more off.*

T: *Okay so we split the 7 into a 5 to count back and then another 2 to get 28. Any other ways?*

Jo: *Take off 10 and add on 3.*

T: *Okay, take off 10 and then add on 3. Next question: 'Wendy wants 50 stamps. She has 17. How many more does she need?' How was that question different to the first two?*

Pat: *I did it as an add.*

T: *You did it as an add? So what made you add that question where you may have subtracted on the first two? What was it that was crucial about that question which meant that adding was easier than subtraction? What made you add rather than subtract?*

Pat: *I know that 17 add 3 was 20, and 20 add 30 was 50.*

T: *So you counted up in two steps almost like imagining a number line. Counting on 3 and then counting on however many tens you need to jump on. Did anyone do it as a subtraction? Did anyone actually take the number away from 50? Arthur how did you take 17 from 50.*

Arthur: *I took 17 away from 50 by just knowing it, by just doing it, by taking 50 minus 40 and then minus the 7.*

T: *So you did 50 take away 10 to leave you with 40 and then you took away 7. Another slightly different way?*

Alex: *Take away 20 and add on 3.*

T: *Take away 20 and add on 3.*

Games can be a good way to encourage children to think about subtraction and to share their methods of calculation with each other. For example *I have, who has?* is a game which can be adapted to suit a range of group sizes and abilities. A miniature version is presented in figure 1.8. In essence you have the same number of players as you have cards. Thus there would be three players for Figure 1.8 – which would be a very small number for a typical game. Each player has a card and player A begins the game by saying, ‘Who has the difference between 5 and 3?’. Player B might then respond, ‘I have 2. Who has 8 minus 5?’ and then player C might say, ‘I have 3. Who has 10 subtract 6?’ and so on. Following each response there is an opportunity for the teacher – or indeed another child – to say, ‘I disagree. How did you calculate that?’. Using such a strategy every time allows pupils to check each other’s answers and demonstrate their thinking. An example of this game[4] is shown on the DVD (Items 1.2a and 1.2b).

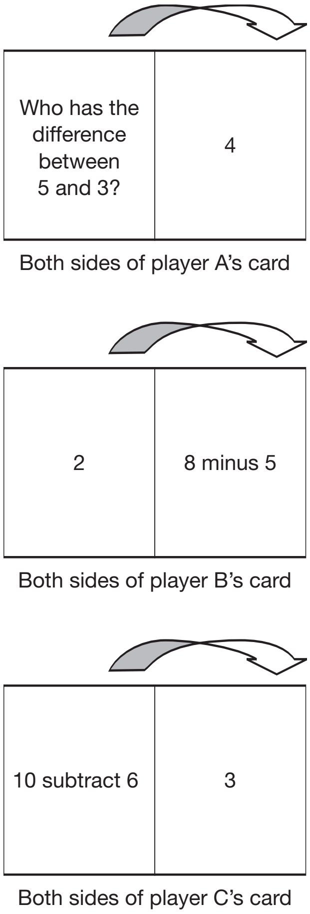


Figure 1.8 *A miniature version of* I have, who has?

A similar idea is to make use of a bingo game. Working with a partner, each pair of children is given a card with a range of numbers on it as illustrated in Figure 1.9a. (The numbers on each of the cards differ and, for example, you may wish to arrange it so that the more confident children have higher numbers which correspond to the more challenging questions.) The ‘caller’ – usually the teacher in the first instance – has a set of questions, a possible sample of which is given in Figure 1.9b. The caller asks a question which each pair consider and discuss. If the answer to the question features on their card then they may cover it up. The first children to cover all the numbers on their card are the winners. The accompanying DVD (Items 1.3a, 1.3b and 1.3c) shows a small group playing Bingo.

4	10	5	9	8
7	3	4	0	2
1	0	5	10	6

Figure 1.9a *A sample bingo card*

4 subtract 2?
What is the difference between 0 and 10?
If I started with 6 cakes and ate 2 of them, how many would be left?
7 minus 3?

Figure 1.9b *Sample Bingo questions*

Working in a classroom recently, I saw two very delighted children when they made the discovery that subtracting the following pairs of numbers always produced an answer of 10: 41 subtract 31; 29 subtract 19; 95 subtract 85. The revelation came about when they were set some questions and invited to use a 100 square, such as shown in Figure 1.10, to help them. It was a real pleasure to see how a relatively simple exercise could give two low attaining children insight into both subtraction strategies and place value.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 1.10 *A 100 square*

Unfortunately some teachers are not so lucky in their choice of resources. For example some people argue that money is a useful way to teach subtraction. They set up shops to encourage young children to buy and sell things and introduce the idea of giving change as a means of practising subtraction. While the idea is perhaps reasonable from an adult perspective, some children can quickly become confused. To a certain extent this is because they usually see their parents using credit cards to purchase items in shops rather than cash. More pertinently here, however, is the fact that some children are introduced to subtraction before they have a sound concept of numbers and what they represent. For example the children who tend to enjoy role play – such as four- and five-year-olds – may view a 10 pence coin as worth '1' rather than representing 10 pence. Similarly they might see a £1 coin as worth '1'. Thus if they paid for an item costing 10 pence with a £1 coin they would not predict the need for any change to be included as part of the transaction. Obviously with older pupils who have a firmer concept of number this is unlikely to be a major issue, although for these pupils, the time for playing shops has usually long gone.