Introduction

> **Pause for thought**
>
> Do you know anyone who feels neutral about maths?

We suspect not. Indeed, we would go so far as to suggest that the majority of people either love it or hate it. Whether we like it or not, however, we are surrounded by numbers and most find it hard not to engage with them several times a day when we phone a friend, search for a room number, buy a newspaper and so on.

Many factors influence how we feel about the subject. Some we will be aware of, but others may be less obvious. Many will be able to conjure up a primary teacher who instilled in them a great love or loathing for the subject. Some will remember bafflement and a feeling of isolation followed by a lack of confidence while tackling a problem, while others will recall the sheer exhilaration of encountering a challenging mathematical investigation. You may think back to occasions when you tried to unlock the key to find a pattern or an easy route to a solution: whether you were generally successful in these quests or not will almost certainly have influenced how you feel about mathematics today.

This book is about how people – in this case generally primary school children – think about mathematics: how they try to make sense of it and reasons for some of their successes and misunderstandings. We have focused specifically on number as this plays a crucial role in the earliest years of schooling and often has a major influence on how we feel about mathematics in later life.

This book is also about broadening our understanding of numbers and how we use them. Some of the chapters – Chapters 0 and 1 – assume very little mathematical knowledge, making them accessible to even the most reluctant readers. Others – most notably Chapter 7 – were specifically written for the more mathematically minded but, if taken slowly, provide a wealth of information useful to us all.

This is a book for anyone working – or about to work – with 3–11-year-olds. Most readers are likely to be prospective or experienced teachers working in schools, but some will be teacher educators, parents, teaching assistants or others interested in children's thinking. A few may even pick it up off the bookshelf to see whether it might enhance their mathematical knowledge and understanding. As a team of teachers, teacher educators, mathematicians and psychologists we hope that you will find something of interest. Indeed, our endeavour is that, whatever your knowledge of classrooms and mathematics, this book will increase your understanding of

number, how we learn about it and how to enhance experience of engaging children in what some see as the mysteries and others see as the delights of the subject.

A biographical note

This book was born in Italy when Anne – a psychologist, mediocre mathematician and primary teacher educator – first met Carlo – a professor in advanced mathematics at the University of Parma – and began discussing why 'zero' is plural in English: we say 'one dog' but 'zero dogs', for example. This led us on to the idiosyncrasies of different languages and the wide scope for misunderstandings when children first encounter mathematics – which itself is a complex and challenging language. In a surprisingly short time the idea for an international project on mathematical misconceptions emerged and colleagues from Israel and the Czech Republic were invited to join the team. A bid for funding was submitted to the British Academy, and we began work on 1 August 2006. The team then expanded to include teachers from across the primary age range in each of the four participating countries, making for a highly experienced and diverse group of experts. Much stimulating discussion and debate ensued as we grappled to understand our different perspectives. We are now all much better informed, and this book reflects our very differing areas of expertise and interests. Each chapter has one, two or even three lead authors, but we have all read, commented on and contributed to all of them, making them – we hope – richer than if written independently and in isolation (see below). Having said that, as may be apparent from the following alphabetical listing, all the contributors are experienced writers in their own right:

Anne Cockburn	School of Education and Lifelong Learning, University of East Anglia, Norwich
Sara Hershkovitz	Center for Educational Technology, Tel Aviv
Darina Jirotková	Department of Mathematics Education, Charles University, Prague
Graham Littler	Visiting Professor of Mathematics Education, Charles University, Prague
Carlo Marchini	Professor of Complementary Mathematics, University of Parma
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Fiona Thangata	School of Education and Lifelong Learning, University of East Anglia, Norwich
Dina Tirosh	Professor in Mathematics Education, University of Tel Aviv
Pessia Tsamir	Department of Mathematics Education, University of Tel Aviv
Paola Vighi	Professor of Mathematics, University of Parma

The book is arranged in three parts. The first (Chapters 0, 1 and 2) makes few assumptions about readers' background and expertise and focuses on largely mathematical matters. The second (Chapters 3–6) adopts a more psychological perspective, while the third (Chapter 7) provides a more in-depth examination of the mathematics taught in primary school classrooms. Chapters can be read in any order but were designed in a specific sequence which favours the more conventional approach of starting at the beginning. For example, from time to time, we refer to various tasks which reappear on several occasions throughout the book. At first sight some of the chapters may not look particularly relevant to you because, for example, you teach a different age than that discussed initially or the mathematics looks too simple. Please take a minute, however, to read three or four pages, for one of the major findings of our work was that a broader appreciation of the number work done across the primary age range reduces the likelihood of mathematical misconceptions arising and enhances both the quality and enjoyment of the educational experience.

Three important starting points

As the title suggests, much of this book discusses mathematical misconceptions and, in particular, how they arise and how they might best be managed. It is important to emphasise right from the start that we do not view misconceptions as a 'bad thing'. On the contrary, they often reveal much about children's thinking and how they acquire – or not, as the case may be – an understanding of complex mathematical concepts. Indeed, they can often be used as useful discussion points in lessons as they can highlight pupils' ingenious – albeit possibly incorrect – ways of tackling a problem.

We also wish to stress that we all have a tremendous regard for primary teachers and the way you work so tirelessly with numerous children and a myriad of demands day in, day out. Having worked in schools, we can appreciate where your enthusiasm might come from, but your stamina and ability to multi-task non-stop are another matter! We wish to reassure you, therefore, that this book is not about pointing the finger at anyone but rather about exploring the many ways in which misconceptions may arise.

This brings us to our final starting point. We see young children starting school as highly knowledgeable, creative and imaginative individuals who have had a wealth of experience with numbers before they ever enter the school gate. Below are some cameos of life in nursery classrooms which illustrate these very points.

Cameo 1. The library in the nursery class was displayed as shown below. Sam announced, 'I want 3 as I'm 3', and promptly removed the book which was in the '3' slot – *Animal Treasury* – and replaced it with the book he had just been looking at.

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Cameo 2. Outside, four 3- and 4-year-olds were working with their teacher, who took on the role of the queen bee. Each in turn was given a choice of four or five hexagonal cards. These had a numeral (between 1 and 9) on one side and the corresponding number of dots on the other. The children's task was to select a card, search for the 'flower' with the corresponding number on it and collect the amount of honey (i.e. bricks) specified on their card and the flower.

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Joey picked out a card with the numeral '9' on it. He duly rushed off to the '9' flower, withdrew nine bricks from the bucket and returned to the teacher. He repeated a similar operation to collect '5 bits of honey'. Spontaneously he merged his two collections of bricks and successfully counted the lot.

Cameo 3. A nursery teacher stuck the following numbers in this order on the whiteboard: 3, 1, 2. A child immediately announced, 'Number 1 should go first, then number 2, then number 3'.

Cameo 4. Jack (aged 3) wrote '3', spontaneously counted three building bricks and then pointed to a picture in the play area saying, 'That's 3 little pigs. I'm 3'.

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Cameo 5. Patsy, aged 3, completed her 'bus stop task' by writing '4' on the stop. Without any comment from the teacher she then wrote another '4' followed by the numbers 1 to 4 twice. She counted all the numbers she'd written and announced that there were 10.
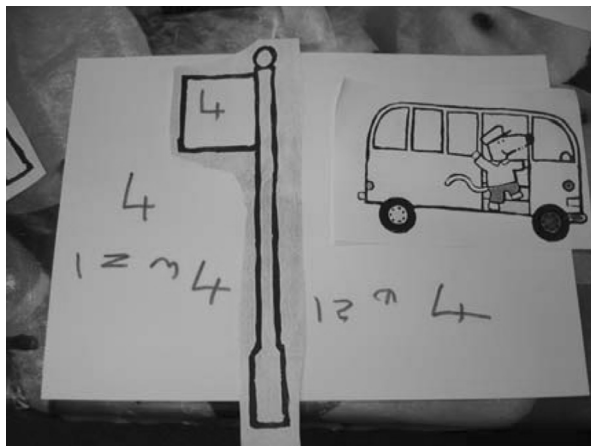
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Cameo 6. Finally, comes from Italy. The children were asked to draw or write a story about the numbers 0, 1 and 10. Gina's thought-provoking response has been reproduced below. For the non-Italian speakers among us 1 is saying to 0, 'You are only nothing'.

Italian picture

Acknowledgements