SPOT THE ERRORS

Identify any obvious errors in rounding numbers or measurements in the following ten statements.

1 The syringe depicted in Figure 8.1 contains exactly 6.5 ml of fluid.

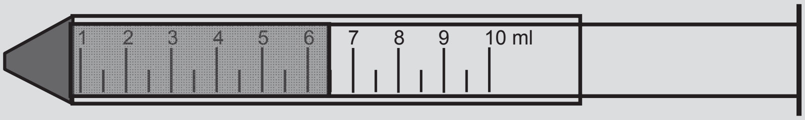

Figure 8.1 How much fluid is contained in this syringe?

2 The syringe depicted in Figure 8.1 contains 6.5 ml of fluid to the nearest half millimetre.
3 The arrow in Figure 8.2 shows the height of a woman on a measuring scale; her height is 157 cm to the nearest centimetre.

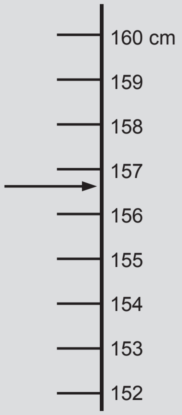

Figure 8.2 Indication of the height of a woman on a measuring scale

4 A patient is prescribed a blood transfusion of 500 ml of blood. This is equivalent to receiving 7500 drops of blood. These are to be delivered over 240 minutes. To find the number of drops to be infused per minute, this calculation is entered on a calculator: 7500 ÷ 240 = 31.25. This result is rounded to 31 drops per minute.
5 An infusion of dextrose 5% has 772 ml remaining. This is now to be infused over the next 6 hours.

Using a calculator, 772 ÷ 6 = 128.666667, so the infusion rate to be set is 129 ml per hour, rounded to the nearest millilitre.

6 In one year the Norfolk and Norwich University Hospitals NHS Foundation Trust treated 729 488 patients. To the nearest ten thousand this number is 730 000 patients.
7 Rounded to three significant digits the number of patients given in Statement 6 above is 729 000 patients.
8 A 50-ml serving of skimmed milk contains 0.0645 grams of calcium; rounded to two decimal places this is 0.06 grams.
9 Rounded to three significant digits the weight of calcium given in Statement 8 is 0.065 grams.
10 A patient is prescribed 3000 micrograms of morphine over 24 hours. Dividing 3000 by 24, a nurse calculates that the patient requires 12.5 micrograms per hour.

Are the errors you have spotted potentially serious in medical health practice?

(errors identified over the page)

Is it possible to make an exact measurement?

In practice every measurement we make is only as accurate as is possible within the limitations of the measuring device we are using, whether it be, for example, a tape measure, a set of weighing scales, a thermometer, or a syringe. You may think when you buy 500 grams of butter, for example, that you have every right to expect to get 'exactly' 500 grams of butter. But if you look closely at the pack you will find a symbol looking like a large lower case letter *e*. This symbol appears on most packages of items sold by weight or volume and is a European Union indication that the quantity can be guaranteed to lie only within certain mandatory limits. It is a reminder that no measurement can be exact. It can only ever be made 'to the nearest something'. Having said that, we will always aim to make our measurements as accurate as is possible within the constraints of our equipment and to record them to whatever level of precision is appropriate.

The limitations of the measuring device

All measurements of such variables as weight, volume and temperature can be only as accurate as possible within the limitations of the piece of equipment being used to make the measurement.

What is measuring 'to the nearest something'?

In everyday life we often round numbers to the nearest something. For example, someone who has an annual salary of £30 864 might say that their salary is 'about £31 000'.

ERRORS IDENTIFIED

The obvious errors are Statements 1, 9 and 10.

Statement 1

What is wrong with this statement is the word 'exactly'. As we explain below, all measurements are in fact approximate; they are always made 'to the nearest something' (as in Statement 2) or to a specified level of accuracy. Statement 1 is not a serious error in medical health practice, although overhearing it may initiate an acute medical condition in a pedantic mathematician (like one of the authors).

Statement 9

The weight of 0.0645 grams is already rounded to three significant digits. The idea of significant digits is explained below. In this case the first non-zero digit reading from the left is the '6'. This is the first significant digit; the 4 and the 5 are the second and third significant digits, respectively. This kind of error would be serious in a medical research context where small variations in recorded data might be important and the third significant digits in measurements are required to register these.

Statement 10

The calculation is clearly incorrect. To see this, round the 24 to 20, just to get a very approximate result using a calculation that is easy to do mentally: 3000 ÷ 20 is equivalent to 300 ÷ 2, which is 150. So we would be expecting an answer in the region of 150 micrograms. This suggests that the result 12.5 micrograms is wrong and the calculation should be done again. The correct result is 125 micrograms. The error in Statement 10 is potentially a serious error in a dosage calculation.

(now continue reading from page 85)

They have rounded the number to the nearest thousand. The salary of £30 864 lies between £30 000 and £31 000, but it is nearer to 31 000 than it is to 30 000. Similarly, a hospital manager might respond to a question about how many consultants are employed by the hospital by saying 'about 350', when the actual number is 339: in this case the manager may have thought it sufficient to round the number to the nearest fifty.

We do the same kind of thing with measurements. A man may say that he is 182 cm tall, when to be precise he should say he is '182 cm tall to the nearest centimetre'. To make this statement there are two steps involved, as illustrated in Figure 8.3:

Two steps in rounding

(a) identify which two points on the scale the number (or measurement) lies between;
(b) decide which of these is nearer to the number being rounded.

(a) identify which of the two centimetre marks on the scale the man's height lies between;
(b) decide which of these is nearer to his height.

In this case the man's height (a) lies between 181 cm and 182 cm, and (b) is nearer to 182 cm than it is to 181 cm.

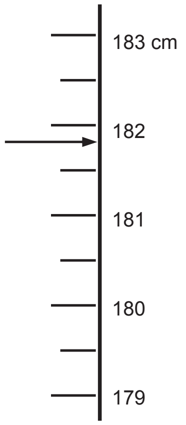

Figure 8.3 Measuring to the nearest centimetre

Rounding to the nearest something

Measurements might be rounded, for example, to

- the nearest whole unit,
- the nearest half a unit (0.5),
- the nearest tenth of a unit (0.1), and so on.

There is a range of values that round to the same value.

Figure 8.3 shows us that any height that falls between 181.5 cm and 182.5 cm would be recorded as 182 cm to the nearest centimetre, because any height in this range would be nearer to 182 cm than either of 181 cm or 183 cm. See also Figure 8.2 and the (correct) Statement 3 in *Spot the errors* at the start of this chapter, where the woman's height is between 156 cm and 157 cm, but nearer to 157 cm. The healthcare practitioner will encounter many situations like these in which numbers and measurements that arise in their practice have to be rounded in some way.

EXAMPLE 8.1

A 12-hour infusion is set up to deliver 1000 ml of a solution of sodium chloride 0.9% (this use of percentages to describe the concentration of a solution is explained in Chapter 13). A nurse calculates that the infusion would require 83.333333 ml to be delivered each hour. (The calculation done on a calculator is: 1000 ÷ 12 = 83.333333.) The calculator result (83.333333) is already truncated to the number of digits that it can display. But, in practice, it is not possible to set an infusion rate to anything like this level of accuracy – nor would it be necessary or appropriate. It is quite sufficient in this example to round this result to the nearest millilitre and to use this, without any risk to the patient's well-being. The result rounded to the nearest millilitre is 83 ml to be delivered each hour.

(We discuss the calculation of infusion rates more fully in Chapter 12.)

The phrase 'to the nearest something' suggests that we are dealing with a spatial idea. It is helpful therefore to have in mind a picture of an appropriate section of a number line, as shown in Figure 8.4. This shows the range of values – between 82.5 ml and 83.5 ml – that are nearer to 83 ml than to any other whole number of millilitres. So, any value in this range is rounded to 83 ml when rounding to the nearest millilitre. The volume 83.333333 ml would lie somewhere between 83 and 83.5 ml, so this is *rounded down* to 83 ml.

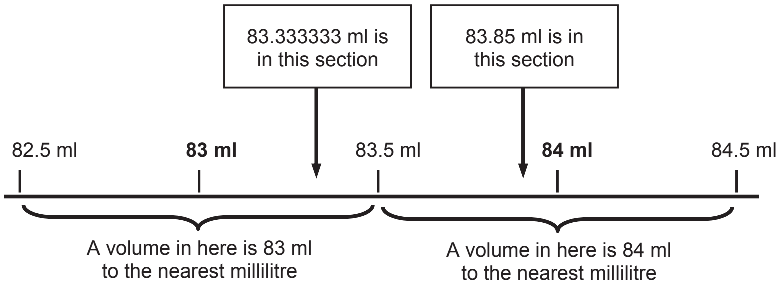

Figure 8.4 Rounding to the nearest millilitre

If, however, a calculation had given an infusion rate of, say, 83.65 ml per hour, then Figure 8.4 shows that this volume is in the range of values that are nearer to 84 ml than to any other whole number of millilitres. So this value would be *rounded up* to 84 ml to the nearest millilitre.

EXAMPLE 8.2

A man attending a diabetic clinic is weighed every six months. The scales being used give the measurement to the nearest half a kilogram (that is, 0.5 kg). On one appointment the digital scales give his weight as 77.5 kg. Six months earlier his weight had been recorded as 78.5 kg. The nurse tells him he has lost 1 kilogram in weight. Is the nurse correct?

No! We will explain. This measuring device is not designed to give a measurement of any greater accuracy than the nearest half a kilogram. So, for example, from 75 kg to 80 kg, the scales can give only these readings: 75.0, 75.5, 76.0, 76.5, 77.0, 77.5, 78.0, 78.5, 79.0, 79.5, 80.0. This is sufficiently accurate; in practice, a change of half a kilogram or less in an adult's recorded body weight is not likely to be of much significance, given variations in things such as fluid retention, the time of day and the weight of socks.

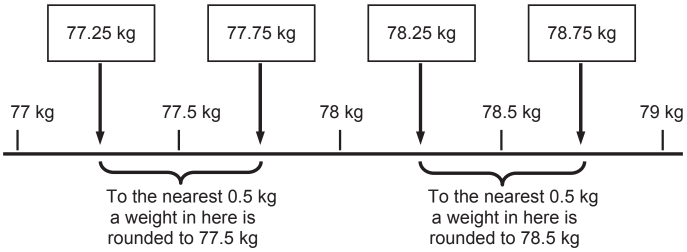

Figure 8.5 Number line showing weights from 77 kg to 79 kg

Figure 8.5 is a section of a number line, showing weights from 77 kg to 79 kg; the scale goes up by half a kilogram (0.5 kg) from one mark to the next. We can see from this diagram that a weight recorded as 77.5 kg to the nearest half a kilogram could be anywhere between 77.25 kg and 77.75 kg. A weight recorded as 78.5 kg could be anywhere between 78.25 kg and 78.75 kg. Figure 8.5 shows that the two weights recorded as 77.5 kg and 78.5 kg to the nearest half a kilogram could differ by any amount between 0.5 kg (78.25 kg – 77.75 kg) and 1.5 kg (78.75 kg – 77.25 kg)! The nurse in Example 8.2 has made the calculation of the change in weight as though the measurements were exact.

EXAMPLE 8.3

A digital thermometer is used every half an hour to measure a patient's temperature. The thermometer being used gives the temperature to the nearest tenth of a degree (that is, 0.1 degrees). Over three hours the readings are: 38.5°, 38.4°, 38.0°, 37.9°, 37.6° and 37.1°. All the readings from this thermometer will have a single digit after the decimal point like these readings. A temperature around 38.44°, for example, would be rounded down automatically by the thermometer to give a reading of 38.4° to the nearest tenth of a degree (see Figure 8.6). This is because 38.44 lies between 38.4 (38.40) and 38.5 (38.50), but it is nearer to 38.4. A temperature around 38.36° would be rounded up to give the same reading of 38.4° to the nearest tenth of a degree. This is because 38.36 lies between 38.3 (38.30) and 38.4 (38.40), but is nearer to 38.4.

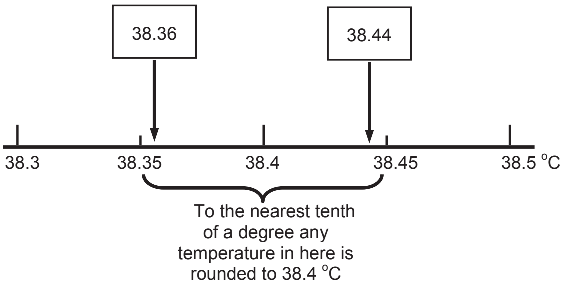

Figure 8.6 Rounding temperatures to the nearest tenth of a degree

Some readers will no doubt be thinking, but what do you do when the number you are rounding is exactly halfway between two possible values? For example, how would you round 67.5 ml to the nearest ml? That's as near to 67 ml as it is to 68 ml. There is no one answer to this question that applies in every situation. In practice, sometimes you will round it down to 67 ml, sometimes round it up to 68 ml; see the next section. If it is practical, just leave it as 67.5 ml.

Is rounding always done to the nearest something?

It is interesting to note that there are situations where you always round up and never round down. For example, you will always round up in any question that asks how many or how much do you need to achieve a target. Here's a simple example of what we mean.

EXAMPLE 8.4

A pharmacy supplies boxes of paracetamol containing 16 tablets. How many boxes would be needed to meet a requirement for 84 tablets? If 84 is divided by 16 the result (5.25) indicates that 5.25 boxes are required. This number will be rounded up to 6 boxes. The context here makes clear that we have to round this result up to 6 boxes, even though the result is nearer to 5 than to 6. With 5 boxes we would fall short of the required 84 tablets.

Similarly, there are situations where you always round down and never round up. If you are aiming to catch the 10.47 train, you would be advised to round this time down (to, say, 10.40) rather than up (to, say, 10.50) in planning to get to the station! You'll be pleased to know, for example, that when the UK tax authorities are calculating your income tax they always round *down* your income to the nearest pound below. Here's another simple example to illustrate this point.

EXAMPLE 8.5

How many doses of 30 ml are available in 500 ml of a solution? If 500 is divided by 30 the result indicates that 16.666667 doses are available. This number has to be rounded down to 16 doses. The context here makes it obvious that we have to round this result down to 16 doses, even though 16.666667 is nearer to 17 than to 16. There is not sufficient for 17 doses.

The point we are making is that the context of the calculation is the first consideration when deciding whether to round up or round down. For example, for an adult patient small additional volumes of fluid arising from rounding up are not likely to be as significant clinically as they would be in a neonate or small child – when it might be safer to adopt a policy of rounding down.

In some cases it is more appropriate to round numbers and measurements to a particular number of decimal places. You may also encounter the idea of rounding to a number of significant digits. These ideas are explained below.

Rounding up or down?

The context that gives rise to a calculation is the first consideration in deciding whether it is more appropriate to round a result up or to round it down.

How do you round to a number of decimal places?

In some contexts the limitations of the measuring equipment or simply a recognition of what level of accuracy is appropriate require that a quantity is rounded to a certain number of decimal places. We described the rounding in Example 8.3 above as rounding to the nearest tenth of a degree. We could also describe this as 'rounding to one decimal place'. This is because each temperature is given with one digit after the decimal place.

Rounding to a number of decimal places

Rounding the number 8.13579 …

- to one decimal place gives 8.1;
- to two decimal places gives 8.14;
- to three decimal places gives 8.136;
- and to four decimal places gives 8.1358.

EXAMPLE 8.6

The infusion rate for delivering 1 litre of a solution of dextrose 5% (see Chapter 13 for the meaning of this 5%) to an adult patient over 6 hours is calculated to be 1000 ÷ 6 = 166.66667 ml per hour. Dividing this by 60 on a calculator, a nurse finds that this is equivalent to 2.7777778 ml per minute. This is rounded to one decimal place, giving a drip rate of 2.8 ml per minute. (We give further explanations of the calculations required in examples like this in Chapter 12.) The equipment being used cannot be set to any greater accuracy than to one tenth of a millilitre– nor would any greater accuracy be necessary in this practical context – in other words only one digit can be used after the decimal place. Rounding the result to one decimal place in this example involves these two steps:

(a) recognizing that 2.7777778 lies between 2.7 and 2.8;
(b) deciding that it is nearer to 2.8 than it is to 2.7.

In step (b) it is helpful to note that halfway between 2.7 and 2.8 is 2.75, and a number starting 2.77… is greater than this.