1 Academic Language

A Foundation for Academic Success in Mathematics

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Mathematic literacy and economic access are how we are going to give hope to the young generation. . . . The idea of citizenship now requires not only literacy in reading and writing, but literacy in math and science.

Robert P. Moses (Civil Rights leader, founder of The Algebra Project)

In today's educational arena, academic language is central to schooling and is one of the most important factors influencing academic success (Francis, Rivera, Lesaux, Kieffer, & Rivera, 2006). The pressures on all students are tremendous, but think about school for English language learners (ELLs) who are trying to learn (and in some cases, relearn) mathematics through a language that they have not yet mastered, English. Teachers are duly challenged in needing to become more aware of the unique features of the language of mathematics to plan instruction that is more strategic and nuanced for content learning. School leaders, in their quest for student achievement, are also realizing that academic language has to be a central curricular focus.

In this first chapter, we define academic language and provide examples of its application to the language of mathematics. We explore how

teachers and students can become aware of language in the mathematics classroom and examine how language is also embedded in the Standards for Mathematical Practice within the Common Core State Standards. Next, we offer a Curricular Framework that serves as the organizing structure for each grade-level chapter, describe each component, and pose questions for teachers or professional learning communities to consider in designing standards-referenced units of instruction. Last, we elaborate on the features of the Framework and their potential uses in various settings.

This chapter is an orientation to all three volumes on the language of mathematics. The examples presented have broad applicability across mathematics topics and grades. This introduction provides a backdrop for the individual grade-level chapters that follow.

WHAT IS ACADEMIC LANGUAGE?

Academic language is a complex concept that can be defined differently by researchers espousing distinct philosophical and methodological perspectives. Although many educators and textbooks refer to academic language as a list of 10 important words for the unit, academic language is more than vocabulary. Academic language or academic English is a *register,* that is, a variety of a language used for a specific purpose and audience in a particular context. Imagine a young attorney talking with a friend at a basketball game as his team is winning the game. Now picture this same attorney speaking in a court of law as he defends a client in a criminal case. The difference in purpose, audience, and context results in clear differences in terms of language use in the selection of words, formality, sentence construction, and discourse patterns.

Broadly, *academic language* refers to the language used in school to acquire new or deeper understanding of the content and to communicate that understanding to others (Bailey & Heritage, 2008; Gottlieb, Katz, & Ernst-Slavit, 2009; Schleppegrell, 2004). In other words, academic language is characterized by the specific linguistic features associated with academic disciplines, including discourse features, grammatical constructions, and vocabulary across different language domains (listening, speaking, reading, writing) and content areas (language arts, mathematics, science, and history, among others). Academic language operates within a sociocultural context that lends meaning to oral or written communication. Figure 1.1 presents a list of different dimensions of academic language along with some examples.

Academic language is also developmental in nature, with increased complexity and sophistication in language use from grade to grade and specific linguistic details that can be the same or vary across content areas

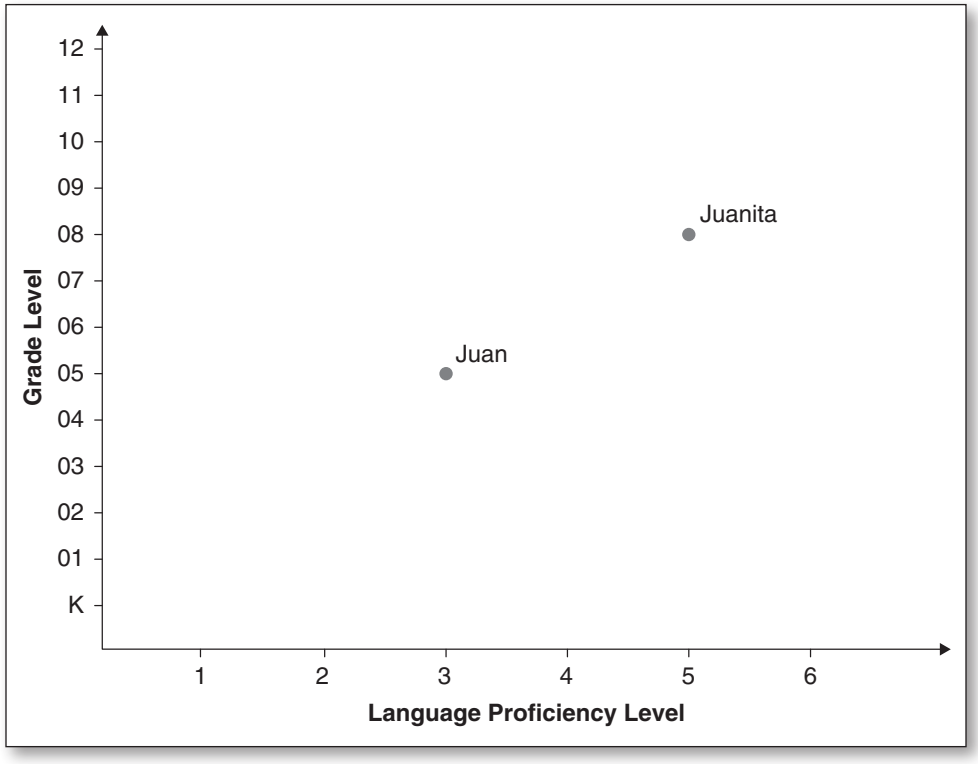
Figure 1.1 Dimensions of Academic Language

Academic Language	*General Areas of Coverage*
Discourse Level	• Text types • Genres • Voice/perspective • Cohesion across sentences (e.g., through connectors) • Coherence of ideas • Organization of text or speech • Transitions of thoughts
Sentence Level	• Types of sentences—simple, compound, complex, compound–complex • Types of clauses—relative, coordinate, embedded • Prepositional phrases • Syntax (forms and grammatical structures)
Word/Phrase Level	• Vocabulary—general, specialized, technical academic words and expressions • Multiple meanings of words • Nominalizations • Idiomatic expressions • Metaphors • Double entendres

(Anstrom et al., 2010). Lev Vygotsky (1987) saw the fundamental difference between the language a child masters by the age of six, and the many long and hard years of study needed to master academic language and concepts that school students face, right up to and beyond the writing of a senior project, a master's thesis, or a doctoral dissertation. Those academic and professional uses of language do require conformity to elaborate, explicit, and often quite mysterious sets of rules. But they are not linguistic rules *per se;* rather they are rules of the academic or professional games (Van Lier, 2012). If we want to participate in these games, we must follow the rules for academic language usage.

As we introduce in *Academic Language for Diverse Classrooms: Definitions and Contexts* (Corwin, 2014), academic language is developmental for all students, increasing vertically from grade to grade, year to year. For ELLs, academic language has an additional developmental dimension, increasing horizontally from one language proficiency level to the next. Note in Figure 1.2 how grades K through 12 are displayed on the vertical axis and language proficiency levels 1 (the lowest) to 6 (the highest) on the horizontal one. Now try to mentally draw the amount of academic language required of Juan, a fifth grader at language proficiency level 3; now compare it with that of Juanita, an eighth grader at language proficiency level 5. What conclusions can you draw?

Figure 1.2 The Developmental Nature of Academic Language for ELLs



The mere growth of academic language associated with each grade level, and proficiency level for ELLs, is not enough to explain its complexity. Also to be taken into account is the building of different types of awareness associated with language use within classrooms.

Awareness of Academic Language

Academic language is more involved than terms, conventions, and genres. In other words, the teaching and learning of academic language requires more than learning about a variety of linguistic components. It encompasses knowledge about "ways of being in the world, ways of acting, thinking, interacting, valuing, believing, speaking, and sometimes writing and reading, connected to particular identities and social roles" (Gee, 1992, p. 73). Put another way, language needs to be understood in relation to the speakers involved, the purpose of the communication, the audience, and the context.

This situated nature of language is integral to mathematical learning. Moschkovich (2002) proposes a situated sociocultural view to describe the language to successfully navigate instructional activities in the mathematics classroom. That is, language is one of the several resources students

need and use to participate in mathematics thinking and learning. Students also draw on social cues (e.g., gestures) and material resources (e.g., artifacts) as well as the use of their home languages to access and construct meaning as they engage in learning.

Language operates within a sociocultural context, not in isolation. In school, the classroom environment often serves as the sociocultural context for learning academic language. Although the distinct backgrounds, experiences, and views of the students need to be taken into consideration, the classroom becomes the mediator for accruing individual knowledge that leads to shared meaning. Thus, by listening to and coming to understand other perspectives, students become a community of learners with its own cultural practices and social norms. In this volume, we come to see distinct communities of practice within each mathematics classroom, each with established social and cultural ways of being (Lave & Wenger, 1991).

There is also a growing awareness on the part of teachers and students of the various processes involved in language learning. Besides the sociocultural dimension that permeates the classroom, students are becoming more conscious of how they learn, and teachers are becoming more responsive in how they teach. Figure 1.3 offers teachers ideas about how to tap students' metalinguistic, metacognitive, and sociocultural awareness within the classroom context.

Figure 1.3 Building Awareness of Academic Language

Type of Awareness	*Classroom Examples*
Metalinguistic Awareness	• Recognizing and identifying cognates in multiple languages • Comparing the similarities among and differences between forms and structures in multiple languages • Transferring information and literacy practices across languages
Sociocultural Awareness	• Using language and culture as resources • Considering and incorporating the students' cultural norms and traditions • Being aware of the situation or context for language learning
Metacognitive Awareness	• Reflecting on how students learn language • Talking and writing about language learning • Discussing with learners about how they do things in the classroom, such as their comprehension strategies

Having established that classrooms are very specialized environments for content and language learning, we now focus on the relevance of academic language in the teaching of mathematics to diverse learners.

The Language of Math

Communication is integral to mathematical learning. Hence, the development of the mathematics register is essential for students to successfully participate in the mathematics classroom. Most teachers will agree that students are required to understand and appropriately use academic English (if not another language) if they are to engage in the teaching and learning process. Until recently, many newcomer ELLs were placed in mathematics classrooms without any instructional support due to the belief that students could readily understand numbers and operations without having to use much English. With the shift in practice to language-focused mathematics, sparked in part by the Common Core State Standards, that kind of thinking has changed.

Many educators consider mathematics a universal language partly because mathematics uses a set of symbols to express ideas using conventional English syntax. Read the following:

$$3\,(4 - x) + (2 + 8) = 64$$

The equation above can be read as "three times four minus x plus two plus eight equals sixty-four." However, to solve this equation, one has to approach it differently than when reading a text in English, such as a newspaper. In English you read from left to right. In mathematics you have to first complete the computations within the parentheses or brackets before you can multiply the first number on the left. Also, difficulties may be encountered when ELLs attempt to translate a mathematical concept expressed in words into a concept expressed in symbols. For example, the algebraic phrase

$$\text{the number } x \text{ is three less than the number } y$$

is often literally translated into

$$x = 3 - y$$

when it should be

$$x = y - 3$$

The two examples above are just a few of the several that will be used when discussing different dimensions of the academic language in mathematics. For students to be successful in the mathematics classroom, they will need to understand and produce the mathematics register. Figure 1.4 provides pertinent examples of the different dimensions of academic language as applied to mathematics.

Figure 1.4 Dimensions of Academic Language

Academic Language	*Examples From Mathematics*
Discourse Level	story problems, calendar math, properties of polygons
Sentence Level	multiplication sentence, comparisons, equations
Word/Phrase Level	altogether, cube, pattern blocks, number line

Discourse Level

Discourse refers to the larger bodies of language—their organization and how they are both coherent and cohesive. Examples of discourse in different content areas include lab reports for science, autobiographies for language arts, and speeches for social studies. Different forms of discourse are categorized into genres. The term *genre* is used to refer to any category of literature or other forms of art or entertainment. For example, in film theory, one basic distinction in genre divides films between fiction and documentary. In education, most are familiar with the different genres required in language arts classrooms, such as historical fiction, tall tale, essay, biography, and public address, among many others. In today's schools, students in kindergarten—and even earlier—begin to recognize and construct different kinds of narratives.

Unfortunately, discussions about different kinds of discourses during mathematics instruction happen very seldom. Story problems or word problems are a kind of discourse used pervasively in elementary and middle school mathematics instruction. Yet, research indicates that little instruction is given to students in terms of how to "read" story problems. In fact, most elementary students approach "story problems" as they would approach a "story" in their language arts classroom. With a story, if students cannot understand a word, they can still get the gist of the meaning from other contextual clues; they can look at the title, pictures, previous sentences, and passages. Those clues do not really work nor are readily available with mathematical word problems. In addition to the lack of contextual clues, there are additional aspects that make story or word problems difficult, especially for ELLs. Figure 1.5 presents five confusing aspects with specific examples.

Sentence Level

At the sentence level, there are language patterns and grammatical structures that are unique to the language of mathematics (Slavit &

Figure 1.5 Challenges of Word Problems

Confusing Aspects of Word Problems	*Examples*
1. Lack of background knowledge The term *sea level* is an abstraction; it can apply to places that are nowhere near the ocean, and places that are below sea level are not literally under water. The phrase *how far apart* does not specify horizontal distance or vertical distance.	A submarine is 285 feet under the surface of the ocean. A helicopter is flying at 4,500 feet above *sea level.* Given that the helicopter is directly above the submarine, *how far apart are they?* (cited in Bielenberg & Fillmore, 2004/2005)
2. Same pronoun is used to refer to different subjects	Suppose *you* and three friends buy a large pizza. *You each* pay with a $5 bill. The pizza costs $12.75. *You* will also pay $0.83 tax on the pizza. How much change will *you and your* friends get? (Houghton Mifflin's *Math Central,* 2001, p. 287)
3. Unclear directions Will the friend start on pane 1 of window 1 or on pane 3 of window 1? Will he or she then move to window 2 or continue in window 1? Is the comparison among the three friends or between the "one friend" and the "two of you"? What is a "pane"?	There are four windows in the attic, and each window has eight panes of glass. One friend cleans every third pane. Two of you clean the rest. Who cleans the least number of panes? (Houghton Mifflin's *Math Central,* 2001, p. 241)
4. Use of several small words These commonly known words (e.g., "time in seconds") take a specialized meaning that is particular to mathematics.	A rock is dropped from a height of 200 feet. During its fall, the rock's height h (in feet) *is given by* $h = -16t^2 + 200$ *where t* is the time in seconds. Find the height *when t* = 0, 1, 2, 3, and 3.5 seconds. When does the rock hit the ground? (McDougal Littell's *Math Concepts and Skills,* 2007, p. 623)
5. Shifting subject referents	Your grandmother started *a college fund* for her grandchildren 15 years ago with an investment of $15,000 at an annual interest rate of 6½%. Find the balance of *the account* if the account earns simple interest. (McDougal Littell's *Math Concepts and Skills*, 2007, p. 378)

Source: Adapted from Egbert & Ernst-Slavit, 2010.

Ernst-Slavit, 2007). These may be highly complex and are encountered primarily in math textbooks. As a result, many students try to literally translate a mathematical concept expressed in words into a concept expressed in symbols. ELLs can often be misled when they attempt to read and write mathematical sentences in the same way they read and write everyday written text. Dale and Cuevas (1992) demonstrated this type of linear, one-to-one translation with the algebraic phrase "the number a is five less than the number b." They found students tended to represent this phrase with the mathematical expression $a = 5 - b$, when it should be $a = b - 5$.

The use of logical connectors (e.g., *consequently, if . . . then, that is, because, such that, but, if and only if, either . . . or, however*) may have very different mathematical meanings than in everyday English writing. In mathematics, these logical connectors may signal cause/effect, reason/result, logical sequence, similarity, contradiction, or chronological sequence. In a 2007 article, Suzanne Irujo outlined how particular sentence structures may affect meaning construction. Figure 1.6 presents a summary of those findings.

Figure 1.6 Selected Examples of Complex Sentence Structures in Mathematics

Sentence Structure	*Examples*
Passive Verbs In passive sentences, the thing receiving the action is the subject of the sentence and the thing doing the action may or may not be included near the end of the sentence.	1. Which number is represented by the shaded part of the figure? (A drawing shows a circle with six parts; four are shaded.) An ELL student may think, "No number is by the shaded part of the figure." 2. Which is read one million, five thousand, seventy-nine? (The question is followed by four choices, one of which is 1,005,079.) An ELL student may wonder, "What *which* are they talking about? Do they want to know which person is reading this number? 3. How much change should she have received? (Question asked at the end of a word problem about buying something and getting the wrong change.) An ELL student may ask, "Does this mean how much change she has, or how much change she gets?"
***If* Clauses** *If* can have very different meanings, depending on the context.	1. *If* you multiply 8 times 5, the answer is 40. 2. *If* I take one counter out of the bag, the probability that it will be red is 1 in 2 (assuming there are 3 red counters, 2 blue counters, and 1 green counter in the bag). 3. *If* we had cut the pizza into 8 pieces instead of 4, the pieces would have been smaller.

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Sentence Structure	*Examples*
Prepositions Multiple prepositions can be used in the same syntactic slot.	6 divided *by* 12 is 1/2 (or 0.5). 6 divided *into* 12 is 2.
	6 multiplied *by 12* is 72 (signals multiplication) 6 exceeds 12 *by* 6 (signals subtraction)
Lack of correspondence between symbols and the words they represent Students take each word literally rather than thinking through what it means in relation to others.	1. In a dictation, students are asked to write the division problem *648 divided by 8.* If they write it *648 ÷ 8,* they should do fine. If, however, thinking that 648 is a large number, they choose to write it as a long division problem, they will very likely write a problem that is read as *8 divided by 648.* 2. In an algebra problem, students read the phrase, "The number *x* is 10 less than the number *y.*" They know they have to write an equation in order to solve the problem. So they write *x* ("the number *x*") = ("is") 10 − ("10 less than") *y* ("the number *y*"). They never realize that the resulting equation, $x = 10 - y$, should really be $x = y - 10$.
Long, complex sentences Mathematical texts often use several phrases and clauses within the same sentence.	In a problem about how many 7- to 10-minute speeches could be given in a 2-hour class, students are asked, "Which of the following is the *best* estimate for the total number of student speeches that could be given in a 2-hour class?" This question contains a complex noun phrase, which contains a complex prepositional phrase, which contains a relative clause construction, which contains a passive construction. (Bielenberg & Fillmore, 2004/2005).

Source: Adapted from Irujo, 2007.

Word/Phrase Level

The language of mathematics has a unique register that includes diverse words, phrases, and expressions. While most educators are aware of the unique vocabulary that differentiates mathematics from other content areas, there are many high frequency words as well as specialized vocabulary that may hinder understanding for all students, particularly ELLs. One way to conceptualize the vocabulary in mathematics is in terms of categories: general academic words, specialized academic words, and technical academic words (e.g., Gottlieb, Katz, & Ernst-Slavit, 2009). Figure 1.7 gives definitions and examples for each of these categories.

Recent studies point to the importance of instruction of general and content-specific vocabulary, including the multiple meanings of words within and across content areas, and explicit teaching of word morphology (structure of words) and word origin (Anstrom et al., 2010). For example, many general academic words like *table, column, mean, product, operation,*

Figure 1.7 Categories of Academic Vocabulary and Examples

Words/Phrases	*Definition*	*Examples*
General Academic	Words, phrases, and idiomatic expressions that are not directly associated with a content area	assessment worksheet pencil in
Specialized Academic	Words and phrases that are associated with a specific content area	odd number equal decimal point
Technical Academic	Words and phrases that are associated with a particular content area topic	hour hand centimeter place value

reduce, expand, and *set* have unique meanings in mathematics different from their everyday meanings. Making sure that students are aware of these multiple meanings will aide in their understanding while avoiding confusion.

Below is a conversational segment from a fourth grade classroom with a large number of second language learners (Ernst-Slavit & Mason, 2011). In this case, the teacher emphasized the multiple meanings of the term "mean" during a review of measures of central tendency because she was aware of how her ELLs could get confused with homonyms, as well as with homophones and synonyms.

56	T	How do we find out **mean?**
57		That's another one of those multi-**mean**ing words
58		isn't it?
59		Am I talking about an attitude when I'm talking about
60		the **mean** for numbers?
61	Students	No
62	T	Whether someone is nice or **mean—**
63	Students	No
64	T	Is that what I'm talkin' about this?
65	Students	No
66	T	What am I talkin' about . . . I'm looking for the **mean** value
67		(pointing to a student from the Ukraine)
68	T	Diana
69	Diana	Um . . . the **mean** . . . um how you find the **mean** you uh the numbers the
69		numbers in your [xxx] you add them up and um and then you di . . . divide
70		divide it by the numb the people that [xxx], and then and then you find
71		the **mean**.

Another area that poses challenges for many students is the large number of multiple terms that refer to the same mathematical operation. A case in point is the many words used to signify addition: *plus, add, combine, altogether, sum, increase, in all,* and *total.*

Although the vocabulary in mathematics is vast, instruction that is deliberate, systematic, and contextualized is necessary for students to engage in mathematical thinking and learning. In addition, students need opportunities to show how they are processing the language they are using in the mathematics classroom. Often students make mistakes not because they do not know what operation to do, but because they misunderstand the meaning of one single word. This is an issue not only for ELLs or struggling readers but for all students. The example below comes from a research study by Celedón-Pattichis (2003) where a student whose home language was Spanish did not correctly solve the following problem:

> On Saturday, 203 children came to the swimming pool. On Sunday, 128 children came. How many more children came to the pool on Saturday than on Sunday?

In this case, it is easy to assume that the *how many more* construction caused the student to misinterpret the problem. However, the think-aloud uncovered the student's misunderstanding stemmed from *than,* interpreted as *then.* More specifically, if the student thinks that the problem is asking how many of the students who came to the pool on Saturday also came on Sunday, the problem will be solved differently.

This short story problem underscores the critical role of language in interpreting what to do mathematically. There are many aspects of mathematical thinking and doing that rely on language that pose challenges for language learners. Figure 1.8 notes some of these language- and culture-related issues encountered by ELLs in the mathematics classroom.

Figure 1.8 Potential Challenges for ELLs in the Mathematics Classroom

Area of Challenge	*Explanation*
Mathematical Thinking	In many countries, school mathematics emphasizes calculations, not communicating mathematical thinking.
Manipulatives	Many students have never seen or worked with math-related manipulatives and might not take a lesson using manipulatives seriously.
Notations	In some countries, periods are used instead of commas when representing large numbers, and commas instead of periods when representing decimal numbers.

Area of Challenge	*Explanation*
Measurement System	Some ELLs are familiar with the metric measurement system and are not familiar with measurements like feet, pints, miles, or ounces.
Reading Mathematically	Students must read mathematics not only from left to right, but also right to left, up and down, and diagonally (in the case of tables, diagrams, and graphs).
Rote Memorization	Some students are used to learning mathematics facts and formulas by rote memorization.
Word Problems	In some countries, word problems are not introduced until the upper grades.
Different Math Curricula	Estimating, rounding, and geometry are first taught in the upper grades in many countries.
Unique Terminology	There are many distinct vocabulary terms used only in mathematics.
Conceptual Density	Mathematics textbooks are tightly packed with concepts.
Place Value	Number name systems in Japanese, Korean, and Chinese provide an advantage in understanding place value and the base-10 system.

Source: Adapted from Egbert & Ernst-Slavit, 2010, pp. 121–122.

Moving from definitions and examples of the academic language of mathematics, we now turn our attention to how academic language is reflected in mathematical practice and what teachers can do to integrate language into content.

ACADEMIC LANGUAGE IN THE STANDARDS FOR MATHEMATICAL PRACTICE

The intent of the Common Core State Standards (CCSS) is to define grade-level expectations that lead to the preparation of students for college and careers. How do we reconcile this goal with the fact that, at the same time, mathematics is considered the gatekeeper to higher education? Knowing that ELLs are not achieving at the same levels in mathematics as their proficient English-speaking peers (see Rampey, Dion, & Donahue, 2009, for example), nor can they be expected to when the language of assessment is English, what can we do as educators to ensure that the gate to higher education is not locked shut for them? All the more reason for promoting the use of academic language in mathematics classrooms!

Let's begin our discussion with the academic language within the CCSS. The Standards for Mathematical Practice describe ways in which

students come to increasingly interact with content as they grow in mathematical maturity and expertise throughout their schooling (CCSS, 2010). The development of Mathematical Practice works in concert with the mathematics standards in supporting instruction.

The Standards for Mathematical Practice encourage student engagement in mathematical thinking and doing as students strive to become mathematics practitioners. To promote mathematical practice, teachers are encouraged to present mathematical tasks that stimulate language use and allow for more than one plausible strategy or solution. Figure 1.9 offers some ideas as to how teachers can use academic language as a means of illustrating each Standard for Mathematical Practice.

Figure 1.9 Promoting Academic Language Development in Implementing the Standards for Mathematical Practice in the Common Core State Standards (CCSS)

Standard for Mathematical Practice in the CCSS	*What Teachers Can Do to Promote Academic Language Development*
1. Make sense of problems, and persevere in solving them.	• Allow students to use their home language to clarify what is being asked • Provide adequate time for students to process the language and content • Encourage working in small groups or with partners • Offer descriptive and timely feedback to students
2. Reason abstractly and quantitatively.	• Provide manipulatives and other resources for students to use to make and interpret mathematical representations • Ask questions regarding students' mathematical thinking • Have students share their reasoning with their peers in English and their home language
3. Construct viable arguments, and critique the reasoning of others.	• Have students engage in peer assessment • Have students listen to and speak, read, and write about different approaches to problem solving • Show how to represent plausible mathematical arguments by modeling or through example
4. Model with mathematics.	• Provide comprehensible input (speech and strategies appropriate to students' proficiency level) for students so they feel comfortable with the academic language of mathematics • Connect mathematics to everyday situations • Use a variety of symbols to connect mathematics terms with ideas and concepts
5. Use appropriate tools strategically.	• Model or demonstrate strategic use of tools in think-alouds • Have students describe mathematical tools and their uses • Encourage students to suggest the use of (nontraditional) tools from their home cultures

Standard for Mathematical Practice in the CCSS	*What Teachers Can Do to Promote Academic Language Development*
6. Attend to precision.	• Emphasize precise use of language to communicate procedures and solutions to problems • Help students develop metalinguistic awareness as it pertains to specifics of mathematics • Balance mathematical accuracy with the need for precision with respect to language
7. Look for and make use of structure.	• Provide opportunities for students to discuss different mathematical patterns that emerge from problem solving • Offer time to explore and examine mathematical structures • Explicitly recognize structural ideas and label them with appropriate mathematical language (e.g., *communicative property,* or *All polynomial functions are continuous.*)
8. Look for and express regularity in repeated reasoning.	• Have students find or produce evidence that validates their mathematical reasoning • Ask students to defend their mathematical reasoning visually, orally, and/or in writing • Honor students' mathematical reasoning outside the anglocentric norm

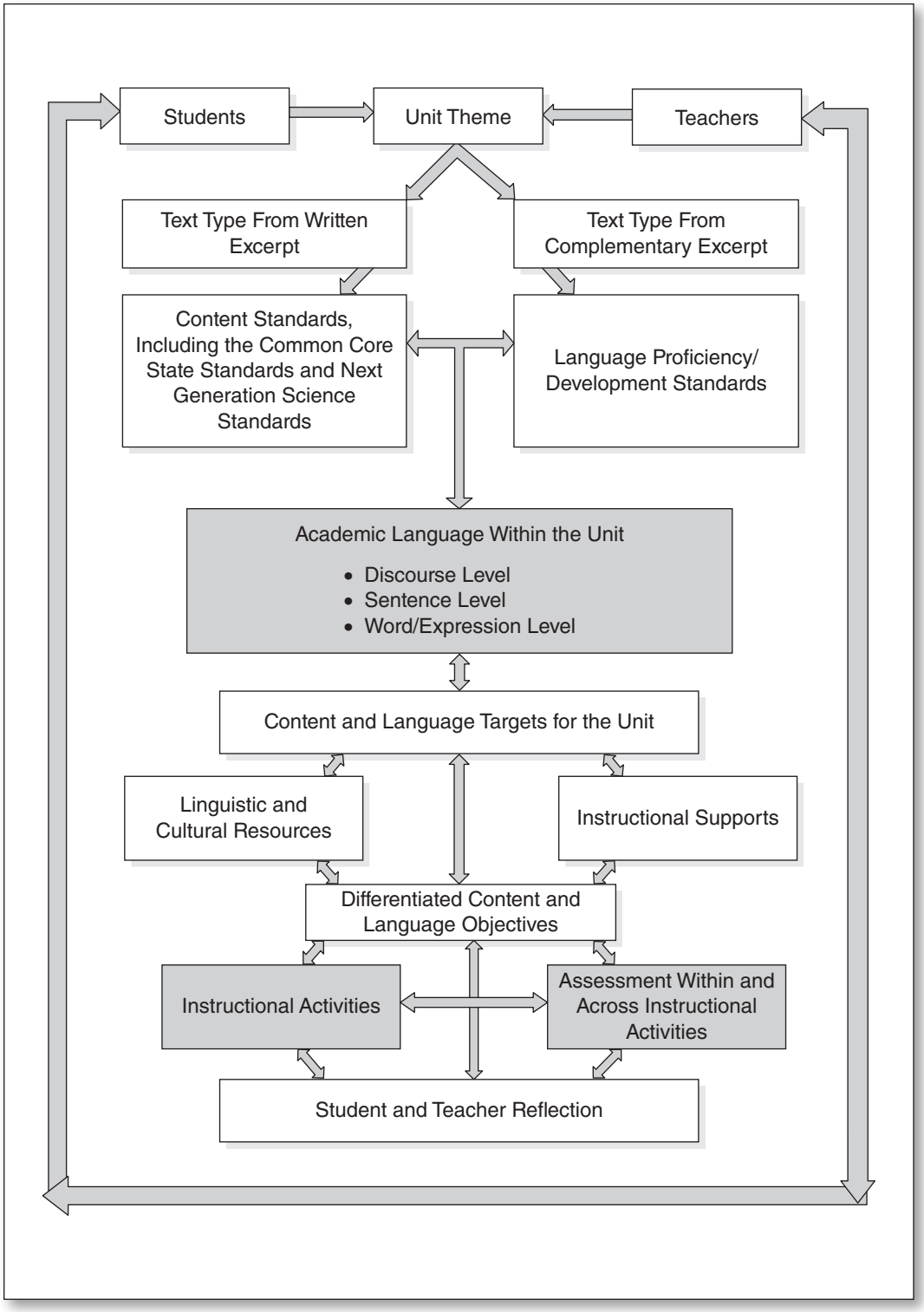
From this discussion on the language of mathematics and its role in standards and mathematical practice, we now turn to the more practical side and explore how academic language can be incorporated into a Curricular Framework for units of study.

A CURRICULAR FRAMEWORK FOR INTEGRATING CONTENT AND ACADEMIC LANGUAGE

Academic language paves the way to academic success, but what does that pathway look like? While there are many ideas of how to incorporate academic language into the school day, we have chosen to integrate language and content in exemplar units of study. Each unit corresponds to a chapter in this volume and represents a connected series of standard-referenced mathematics lessons around a topic or theme being taught at a specific grade level.

The Curricular Framework, shown in Figure 1.10, forms the organizing structure for the chapters in this book and throughout the series. In this introduction, we first describe each component. These descriptions are followed by a set of questions to guide professional learning communities, grade-level teams, or collaborating teacher pairs in thinking about applying the framework to their own settings. We invite you to share, select, and use the questions that are posed as a starting point for designing a linguistically and

Figure 1.10 A Curricular Framework Highlighting Academic Language



culturally responsive curriculum. Appendix A, at the close of this chapter, is a blank template of the Framework. We realize that there are many ways of designing curriculum and honor whatever is currently in use in your school or district; therefore, consider how the components of this framework might be flexible and adaptable to your setting.

In this Framework, we highlight the role of language in learning. As each chapter unfolds, you see how academic language is part of content learning and how content provides the context for language learning. It is the academic language woven throughout the Framework that provides its cohesion and gives teachers a focus for instruction and assessment.

The curricular units are intended for all students; however, we pay special attention to linguistically and culturally diverse students whose interplay of languages and cultures greatly contributes to the classroom and school climates. When we envision grade-level academic language, we are sensitive to where students begin their educational journey, which languages they use along the way, and where they need to go. We set clear targets, invite students to help define and refine the criteria for success, and together reflect and decide next steps.

Many stakeholders—parents, students, paraprofessionals, teachers, school leaders, administrators, and school boards—work to attain educational excellence for their students, schools, and communities. With this series, we recognize the diversity of classrooms throughout the country and how to bring academic language into everyday conversations for all educators and students to partake of and enjoy.

The **Students** are the starting point and central focus for curricular planning and educational decision-making. In any school, students are an eclectic mix of personalities with unique upbringings who come with varied perspectives on learning. The interaction of students among themselves and their teachers forms the vitality of a classroom. In today's diverse classrooms, more and more students represent the myriad of languages and cultures reflective of our global society. Getting to know the whole child—students' linguistic and cultural vantage points as well as their educational and personal histories—should help educators ascertain the students' familiarity with grade-level academic language that, in turn, will help inform how to construct and enact curriculum.

Linguistic and Cultural Backgrounds

- Which languages and cultures do the students represent?
- Which generations of families are living in the household?
- What are the students' or families' countries/regions of origin?

Other Characteristics

- What are the students' personality traits—for example, are they reticent to participate or linguistic risk-takers?
- To what extent are the students motivated to learn and are persistent in learning?
- What family circumstances might facilitate or impede student learning?

Educational Experiences

- Have the students attended preschool?
- Have the students had continuity in schooling from grade to grade?
- If the students are ELLs, have they had continuous language support services from year to year? And if so, what kind and how much?
- What is the students' school attendance from year to year?

Performance

- What is the academic achievement of students in the core content areas?
- What is the oral language proficiency of students in their home language and English?
- What are the levels of literacy of students in their home language and English?
- In which areas, outside the core, do the students have strengths?

Teachers are educational professionals dedicated to continuous improvement of their craft through the teaching and learning cycle. The teacher leaders in the chapters that follow embed academic language within curriculum development and often work collaboratively to reinforce and extend learning to their students. Teachers' specific training, content and language expertise, classroom style, and interactions with students all contribute to their effectiveness in mathematics classrooms.

Qualifications

- In what areas do teachers hold certificates?
- Do teachers have any additional endorsements?
- Do teachers have any special recognition or training?
- Do teachers speak a language other than English on a regular basis?
- Have teachers experienced learning languages in addition to English and dabbling in other cultures?

Experience

- How many years of classroom experience do teachers have?
- Which grade levels have teachers taught and to what extent have they worked with students from linguistically and culturally diverse backgrounds?
- Have teachers collaborated by coteaching, reciprocal teaching, or participating in professional learning communities?
- Have teachers been mentors, team leaders, or coaches to novice teachers working with ELLs?

The **Unit Theme** identifies a grade-level topic of interest that can be pursued in depth from multiple vantage points. It provides the unifying thread for weaving content and language instruction. In particular, the unit theme offers rich opportunities for students to examine and explore academic language tied to overarching concepts that develop over several weeks. In this volume, the primary conceptual focus for the theme is drawn from mathematics, although in many chapters, it is coupled with one from English language arts.

- What topic is relevant and engaging for the students and have they been involved in its selection?
- Does this topic lend itself to rich and deep content and language learning?
- Can this topic readily fold into a multidisciplinary theme that crosses content areas and language domains?
- To what extent is the topic critical for success of all students in this grade level?
- Is the topic grounded in both content and language standards?

Different **Text Types From Written Excerpts** reveal the academic language of mathematics in varying displays and contexts. Students who deal with multiple text types, including informational texts and narratives, gain a broader sense of the mathematical world around them. The use of excerpts from multiple texts in each unit enables teachers and students alike to see the broad applicability of mathematics to the real world.

- Are the texts appealing, age appropriate, and compelling to the students?
- Do the texts represent multiple literacies with a range of genres?
- Do the texts typify grade-level content and academic language for the topic?
- Do the texts illustrate different ways of looking at the topic?

Content Standards, Including the Common Core State Standards, are a driving force in organizing curriculum and identifying the academic language related to the knowledge and skills of the unit's theme. Along with "processes and proficiencies," attending to precise language is very much a part of the Common Core State Standards for Mathematics. Today's mathematics standards insist on a balance among conceptual understanding, procedural fluency, and real-life connection. Mathematics standards extend beyond having students develop a strong understanding of numeracy to having them reason, problem solve, justify, and explain their understandings. In addition, as evidenced in some of the chapters, mathematical practices help support students as mathematicians.

- Which grade-level content standards match the topic or theme?
- Which content standards exemplify grade-level expectations illustrated by the texts?
- Are there related standards that can be drawn within or from additional content areas?
- Do the content standards build or scaffold on students' knowledge and skills?

Language Proficiency/Development Standards are expressions of language expectations designed for students who are on a pathway toward acquiring a new language, in this case, English. Generally descriptive statements, language standards account for how language learners process or produce language for a given purpose and situation (Gottlieb, 2012). In the last decade, language standards have become associated with the academic language necessary for students to access and achieve grade-level content. Some states and organizations use the term *proficiency;* other states and consortia prefer the term *development* to describe this process; therefore, both are recognized in these volumes. Together, content and language standards provide a full complement of projected academic milestones for students.

- Which language proficiency/development standards correspond with the selected content ones?
- Which language proficiency/development standards are illustrated in the selected instructional texts?
- How is academic language represented in the standards?
- To what extent are language expectations differentiated by students' levels of language proficiency?

Academic Language, the language that students navigate and negotiate as part of schooling, is central to standards-referenced curriculum,

instruction, and assessment. It permeates our diverse classrooms and represents the key places in the framework where content and language intersect, such as the unit theme, texts, and targets. We envision academic language operating simultaneously at different levels, from the words and expressions that make up sentences, to the grammatical forms within the sentences, to the organization of text and speech around discourse. Teachers and students need to be aware of the role of academic language in teaching and learning.

- What grade-level academic language pertains to the topic or unit theme?
- What is the academic language embedded in the texts and other instructional materials?
- What is the academic language associated with the concepts and skills of the content standards?
- What is the academic language associated with language standards?

Discourse Level

Each subject area has its own genres that students read and write; each genre, in turn, has its own grammatical and word choices (Schleppegrell, 2004). Discourse in mathematics encompasses the interchange of ideas—both written and oral—that represents ways of thinking and communicating precisely and mathematically. In discourse-rich mathematics classes, students discuss and explain the strategies and processes used in solving mathematical problems and connect their own everyday language with the specialized language of mathematics (Sonoma County Office of Education, 2012).

- How is the language (text or speech) organized and how does it flow?
- Which text types or genres are represented?
- Which elements of the text or speech provide cohesion?

Sentence Level

Mathematics is a school subject area that has its own distinctive grammatical features and language structures that students must negotiate to be successful learners (Abel & Exley, 2007). Sentences that revolve around mathematical concepts tend to have dense noun phrases and precise meanings for conjunctions and prepositions, and they often are phrased in the passive voice, as in *are taken away* or *is divided by* (Schleppegrell, 2004).

- What are some grammatical forms important to understanding the specific text?

- What are some of the sentence types within the text?
- How do the different sentence types lend voice or perspective?

Word/Phrase Level

Mathematical words and expressions need to be presented in meaningful and familiar contexts for students. There are many everyday terms that have specialized meanings in mathematics, such as *power, table, balance,* and *odd,* to name a few (Zwiers, 2008). In addition, there are numerous terms that signal the same operation, as in *multiple, times, double,* and *factor* (Egbert & Ernst Slavit, 2010).

- What are some topic-related words with multiple meanings?
- What are some words that may serve as cognates, particularly for Spanish?
- Which key words or phrases contribute to the understanding of the theme?
- What are some nuanced expressions, idiomatic expressions, or metaphors that enhance the meaning of the topic or theme?

Content and Language Targets represent the overall goals or focal points of the unit that are intended for all students. The content target relates the "big idea," "essential understanding," or primary concept and directly corresponds with the content standard. The language target reflects the language function (the communicative purpose) that best fits this concept for all language learners and is often an expression of the language development standard.

Content Target

- What is the overall content expectation for the unit?
- What are the most critical concepts students need to learn?
- How might this target be illustrated from lesson to lesson?

Language Target

- What is the overall language expectation for the unit?
- What is the most critical language function students need to process or produce to construct meaning related to the content or theme?
- How might this target be practiced from lesson to lesson?

Linguistic and Cultural Resources are plentiful in our diverse classrooms. Each student brings a personalized history with its own cultural

orientation; family members also contribute to each student's linguistic and cultural repertoire. The people, events, and places in the community in which the students reside provide cultural layers. There are also materials and artifacts within the home, school, and community that can be tapped to make connections and extend learning. The importance of building on the knowledge and skills held by students, their families, and the communities in which they reside, or their "funds of knowledge" (Moll, Amanti, Neff, & Gonzalez, 2001) is illustrated throughout the grade-level chapters.

The Students

- How can students' languages and cultures be tapped to enrich the unit?
- What cultural perspectives or cultural capital do the students bring to the topic?
- What do the students know about the topic based on their personal experiences?

The Community

- How can the students' and their families' cultural views and practices contribute to their learning in school?
- What community models or expertise might be useful to reinforce or extend student learning?
- Which community organizations, activities, or events can be tapped as part of curriculum development?

Instructional Materials

- What multicultural materials lend themselves to the learning experience?
- What other multicultural materials might be available?
- How can technology increase linguistic and cultural input into the unit?

Instructional Supports are essential for introducing, reinforcing, and assessing concepts and their accompanying language. Supports invite learning through multiple venues. In the mathematics classroom, ELLs need multiple supports to provide visual, tactile, and interactive scaffolds to learning. Encouraging the use of the primary language enables students to connect with prior knowledge and confirm their understanding of new ideas (Coggins, Kravin, Coates, & Carroll, 2007).

- Which visual or sensory supports maximize scaffolding of language?
- Which graphic supports aid in comprehension?
- How can student–student interaction reinforce language learning?
- What other kinds of interaction promote language learning?

Differentiated Objectives enable all teachers to set realistic expectations from lesson to lesson and allow students to be challenged while actively engaged in learning. Differentiation allows for fit and success for today's diverse learners (Tomlinson, 2001). Differentiated content objectives provide different avenues to acquiring the skills and concepts named or implied in content standards. Differentiated language objectives provide ELLs the means for accessing and achieving the content for their given levels of language proficiency.

- Why are both unit targets and lesson objectives necessary?
- How do differentiated objectives fold into content and language targets?

Content Objectives

- What are the students' conceptual understandings of the lesson?
- What are the students' skills related to the lesson?
- What provisions, if any, are made for ELLs (e.g., use of home language) to show their content knowledge?

Language Objectives

- What are the language expectations for the students' levels of (English) language proficiency for the given lesson?
- How is academic language to be represented at the sentence and word levels?
- How can we ensure that all students are exposed to and have opportunities to interact with grade-level language?

Instructional Activities are the mainstay for implementing academic language in classrooms. Teachers can readily integrate grade-level content and its related language into instruction to make it comprehensible for all students. Innovative activities within lessons should draw from the students' linguistic and cultural resources to enable them to form new learning. Related activities may combine or build upon each other to create tasks and long-term projects.

- How can language and content be integrated into instructional practices?

- How can authentic, engaging, and yet challenging activities be designed?
- Do the activities allow students multiple pathways to show what they know and are able to do both conceptually and linguistically?
- Do the activities involve higher order thinking for all students?

Assessment, built into the instructional routine, produces standards-referenced data that measures the extent to which content and language objectives have been met lesson by lesson. To the extent that instruction is differentiated, so should assessment. To the extent that instructional supports are used for ELLs and other students, so should assessment. To the extent that instruction is centered on academic language, so too should assessment. While day-to-day assessment relies on immediate descriptive feedback to students and meeting short-term instructional objectives, assessment for the unit, across lessons, is comprehensive and in-depth with more thorough measurement of the criteria associated with the content and language targets.

Within Lessons

- What are some checks for student understanding of content and language?
- How is academic language measured for each lesson's differentiated objectives?
- What documentation can be used to show that language or content objectives have been met?
- What kind of descriptive feedback can be provided to each student?

Across Lessons

- In what ways do students demonstrate that they have met the content and language targets?
- What kinds of documentation forms lend themselves to capturing the content and language targets?
- Does the documentation reference or reflect content and language standards?
- How are assessment results reported and reviewed?

Unit Reflection is guarded time for evaluating the teaching and learning processes and to pinpoint the successes and the areas for improvement. Students and teachers are partners in the process. Students

have opportunities to match their expectations for learning to their evidence of learning and to share results with their peers, family members, or teachers. Teachers rely on student feedback to make judgments about the effectiveness of their instructional strategies or use data collected throughout the unit to decide to what extent targets and standards have been met.

Students

- What is the evidence that students are moving toward or have met their criteria for success?
- How might the students describe their performance in terms of their achievement and language proficiency?
- In what ways might peer assessment or student self-assessment contribute to students' content and language understanding?

Teachers

- What instructional adjustments are made based on assessment within lessons?
- What did the teacher or teachers learn from their students?
- What could be done differently next time and why?

Features of the Curricular Framework

The Curricular Framework, in essence, is a long-term planning guide for teacher teams or professional learning communities to ponder, prepare, and put into action. It can serve as an outline to organize thematic instruction around academic language across grade levels or a guide to build consensus around common standards-referenced themes and approaches for integrating language and content in instruction and assessment.

The contributors to this volume have followed the Curricular Framework in planning and implementing their mathematics unit so that you, the reader, are able to envision its consistency of use and applicability to different educational settings. Yet, at the same time, in the individual chapters, to maintain authenticity, we adhere to the terminology used in the various districts or regions, such as language proficiency or language development standards, and the names of the specialized approaches of language instruction, and titles for teacher endorsement or certification. We also refer to the content standards that teachers use in their states, including the Common Core State Standards and the Next Generation Science Standards. Last, we realize that there are many configurations for arranging curriculum and that designing assessment often precedes

instructional activities; again, the contributors have remained true to whatever the school or district uses.

As a means of better understanding the Curricular Framework and how it might be implemented, we wish to highlight and describe its major features.

It's intended for use with all students

There is a tremendous diversity of languages, cultures, competencies, and experiences among our students. While no two students enter school with the same set of life circumstances, all require strong academic language to ultimately succeed. Therefore, we hope that educators realize the broad applicability of the framework to all their students and find it relevant and useful in thinking about and planning for differentiated curriculum, instruction, and assessment.

It's to be synchronized with other school and district initiatives

Over the years, states and school districts have come to use various curricular designs and instructional materials. In some cases, specific approaches or textbook series are mandated; in others, the decision might rest with a grade-level or department team or a professional learning community. Whatever is in place is to be respected; inasmuch the framework offers a suggested sequence of components that compose an instructional unit of study, however, it might be advantageous to see how it might coincide with other curricular resources.

It's adaptable to various settings

Given the range of contributors who have used the Framework to design and implement a unit of instruction, its usefulness is quite evident. The unique classroom contexts described at the beginning of each chapter underscore its applicability to many different teaching situations. In this series, we recognize the variability in the concentration of linguistic and cultural diversity across our nation's schools; although there is a presence of ELLs in each chapter, we realize that this is not the situation in all schools that focus on academic language.

It highlights academic language

More than any other feature, the Framework attempts to emphasize the critical importance of academic language in planning and implementing curriculum, instruction, and assessment. It is so prominent that academic language in and of itself can serve as the principle around which

classrooms are organized. Although we feature the language of mathematics in this volume, we also recognize that academic language permeates all facets of school and schooling.

Its content and language components are paired and aligned

When content and language join forces, both are reinforced and become strengthened. As part of the Framework, we have coupled content standards with language standards, content targets with language targets, and content objectives with language objectives to purposely accentuate how content and language complement and bolster each other to promote learning. As language is the medium through which content is delivered, educators should not think of one without the other.

It begs collaboration between content and language teachers

The pairing of content and language components facilitates discussion about the role of content in language instruction and the converse, how content instruction is mediated through language. The framework places language and content teachers on equal footing with both contributing to and responsible for the education of all students. With teachers working together toward the mutual goals of setting and meeting high academic expectations, students are bound to benefit.

It welcomes sociocultural perspectives by bringing linguistic and cultural diversity to the forefront

Learning is a social experience where interaction among students and between students and teachers is integral to teaching and learning. As students represent a multitude of languages and cultures, there should be ample opportunities in the mathematics classroom for them to express their experiences with content and concepts from their linguistic and cultural vantage points. Their contributions should not only be welcome in building a classroom climate but be incorporated into the study of mathematics.

It extends supports for learning across instruction and assessment

Visual, sensory, graphic, and interactional supports are a means of bringing additional ways of meaning making into classrooms. These multiple ways of representing concepts enable students to access content and engage in higher order thinking irrespective of their level of language proficiency and their school experience. Supports for instruction and

assessment enable students to show what they know and are able to do without total dependence on print or oral language; thus, they provide scaffolding for ongoing language development.

It fosters fluidity between instruction and assessment

Instruction and assessment are introduced side-by-side in the framework, implying that planning for assessment can occur before instruction, during instruction, or after instruction. Contributors to this volume follow the curriculum design that is in place in the schools they are describing, therefore, the book covers a variety of approaches in which instruction and assessment interact. Thus, although instruction and assessment are shown as separate components, when students are engaged in activities based on predetermined criteria and produce original work, instruction and assessment are blurred to become instructional assessment (Gottlieb, 2006).

It encourages reflection on teaching and learning

The Framework acknowledges that teaching and learning are dynamic processes that occur in classrooms where both teachers and students alike take time to think and reason mathematically. Ideally, the metacognitive, metalinguistic, and sociocultural awareness that is being fostered throughout the unit is being systematically shared among teachers and students. For students, reflection might occur in both their home language and English so they can acquire and communicate deep understanding of content and language. For teachers, reflection entails examining evidence for learning, sharing what they discover with students and their fellow team members, and making instructional adjustments based on the information.

REFLECTION ON THE VOLUME: LOOKING BACK AND MOVING FORWARD

At the close of each chapter, contributors summarize the key ideas and show how a class of diverse learners has taken on the challenge of a mathematics unit of study that has been carefully crafted by teachers devoted to their craft. We are optimistic that teachers are able to see themselves in the pages, relate to the dialogue and interaction with the students, and come away energized with new ways of integrating content and language in their classrooms. But above all, we hope that teachers see the value of centering their instruction and assessment around the multiple dimensions of academic language.

APPENDIX A
A TEMPLATE OF A CURRICULAR FRAMEWORK HIGHLIGHTING ACADEMIC LANGUAGE

You are welcome to adopt or adapt this Framework when thinking about how, where, and when to embed academic language within units of instruction. You may use the descriptions of each component outlined in this chapter as your guide or create others with your professional learning team.

Students: Teachers: Unit Theme:

Text Types of Written or Oral Excerpts From Multiple Instructional Materials:

Content Standards, including the Common Core State Standards and Next Generation Science Standards: Language Proficiency/Development Standards:

Content Target for the Unit: Language Target for the Unit:

Linguistic and Cultural Resources:

Academic Language Within the Unit:

Discourse Level:

Sentence Level:

Word/Expression Level:

Instructional Supports:

Visual or Sensory:

Graphic:

Interactive:

Differentiated Content Objectives:

Differentiated Language Objectives:

Instructional Activities:

Assessment Within Activities:

Assessment Across Activities:

Student Reflection Opportunities:

Teacher Reflection Time:

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