2 Grade 6

Exploring Possibilities With Geometric Solids

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Geometric forms have multiple possibilities. That's what's so fascinating about them.

Artist Pancho Quilici (Snow, 2006, p. 21)

CLASSROOM CONTEXT

Content:	Mathematics—Geometry
Grade:	6
Languages of the Students:	Spanish, English
Teacher Experience:	First year
School:	Grades K–8; Rural NW U.S.; 78% free and reduced lunch; high poverty; dual immersion; over 90% Hispanic student population; 40% early advanced or advanced ELL students

Teachers

Claire looked around her math classroom and smiled ruefully. Her Spanish language teaching partner, Eduardo, was about to come in for a data analysis meeting. Although her classroom was normally neat and

organized, at the end of this sunny April day, there were bits of tissue paper on the floor; a pile of brightly wrapped cones, spheres, and pyramids leaning against her desk; and printouts of piñata forms arranged nearby as part of Día de los Niños preparations. Claire couldn't believe she was nearing the end quarter of her first full year of teaching in a public charter school in the inland Northwest.

While frequently feeling a little overwhelmed, Claire had enjoyed her first year at this high poverty, K–8 dual immersion charter school. She and her teaching partner, Eduardo, taught all core subjects—math, science, language arts, and social studies—to fifth and sixth graders. In addition, they taught project-based learning classes (which students planned and teachers guided across content areas) to groups of multiage students every Friday morning. In their bilingual school model, their 28 sixth graders and 25 fifth graders rotate from Claire's English classroom to Eduardo's Spanish classroom each alternate week. One week Claire has fifth graders in English, while Eduardo teaches the sixth graders in Spanish; the next week, Claire has sixth graders and Eduardo has fifth graders, and so on. The curriculum for each grade moved forward, but students' target language alternated each week. Claire and Eduardo's daily teaching schedule is outlined in the chart below. While the schedule was the same for Claire and Eduardo, the languages of their instruction were different (see Figure 2.1). In addition, Claire was careful in planning each lesson; an example of the template she uses is shown in Appendix A.

Figure 2.1 Sixth Grade Student Schedule

	Monday	*Tuesday*	*Wednesday*	*Thursday*	*Friday*
A.M.	Language arts				Project-based learning classes
A.M.	Social studies	Students in art; Claire and Eduardo's planning time	Social studies	Students in art; Claire and Eduardo's planning time	
A.M.	Fourth through eighth grade language development				
P.M.	Science				Health
P.M.	Math				Students in technology; Claire and Eduardo's planning time

Claire loved her students, who seemed to have such deep ties to their school, which had the only bilingual program in their large rural county. She had especially enjoyed working with her students' families; the school promoted strong home–school connections, including home visits and special events for families weekly (like Friday breakfasts) and throughout the year. She knew most families struggled to make ends meet in her county, which was the highest poverty county in the state and tenth in the nation; however, many parents and family members were very involved with the school.

Claire taught the English immersion side for the sixth grade math block. In this setup, she had the school's 28 sixth graders for one week in English immersion before they rotated to her teaching partner, who taught the Spanish component. They didn't reteach each other's lessons; rather, in the immersion setting the two teachers planned units together, knowing that their students' language of instruction simply changed as the students moved from week to week. Most of Claire's sixth grade students had been in the dual immersion school for four years or more, so they had some proficiency in both English and Spanish, although their literacy and academic language competencies varied in each language.

During her preservice teacher preparation program, Claire had not thought about what it would be like to teach in a dual immersion team-teaching context. Luckily, her teaching partner, Eduardo, had already been teaching at the school for four years and was able to provide her with a conceptual understanding of how they would coordinate instruction in order to scaffold student comprehension and transfer academic concepts across both languages. Before she had entered her teacher education program, Claire had taught at-risk English language learner (ELL) teens in a private program in an urban setting, and she had taught adult ELLs outside the United States, so she had some experience with the challenges and joys of language teaching as she taught new content.

Claire's Sixth Grade Students and Their Language Proficiency Levels

Most students in Claire's classroom have attended school together in this small school environment since kindergarten or since their early primary years. Thus, the culture of their schooling experience has a strong emphasis on a sense of community, family involvement, and the principles of Pride, Purpose, and Performance, which are the school's lifelong learning goals. The students are often full of energy and excitement to learn. Claire enjoys the everyday surprises as their personalities and sense of humor emerge throughout the year.

In her dual immersion charter school, Claire knew that most of her sixth graders had developed strong degrees of fluency in both English and

Spanish, with the exception of the more recent arrivals to the program. Nonetheless, it was fairly rare to have a recent arrival from a Spanish-speaking country who had not had previous educational experiences in English. It was more common for students either to come from an English-speaking family, or to come from a Spanish-speaking family but to have had a K–5 education taught entirely in English. Thus, new arrivals often needed more language scaffolding in Spanish than in English.

As she thought about planning her geometry unit, she reviewed her 28 students' language development levels in English. For students still officially categorized as ELLs, she was able to see how they were categorized based on the Oregon English Language Proficiency Assessment (ELPA); then she considered whether the assessment appeared to be an accurate representation of their language competencies in her math classroom. In addition, she collected information about her students from discussion of student progress with other fourth through eighth grade English language development teachers.

Thus, Claire felt confident with her identification of students' linguistic strengths and areas to provide additional support. According to the Oregon English Language Development Standards (Oregon Department of Education, n.d.), her early advanced students "demonstrate consistent comprehension of general meaning; good understanding of implied meaning; sustain conversations, respond with detail in compound and complex sentences; actively participate using more extensive vocabulary, use standard grammar with few random errors." Her advanced students, on the other hand, were able to demonstrate "comprehension of general and implied meaning, including idiomatic and figurative language; . . . initiate and negotiate using appropriate discourse, varied grammatical structures and vocabulary; use of conventions for formal and informal use" (p. 4).

Her remaining students were all proficient English speakers or balanced bilinguals (students identified as being able to switch from one language to the other with similar levels of confidence and proficiency within an environment of sustained academic support in two languages).

English Language Proficiency Levels in Claire's Classroom

Early Advanced ELLs	*Advanced ELLs*	*Balanced Bilinguals or Proficient English Speakers*
Enrique, Javier, Julio	Cloe, Clarissa, Cristian, Rosa, Lilli, Maria, Diana, Carlos	Jenny, Silas, Nina, Geraldo, Fátima, Joseph, Luis, Sofia, Alondro, Flora, Kristen, Michael, Ana, Aliana, Leon, Tim, Penny

Claire believed that learning new mathematics language in English was an important part of teaching math for all her students. But Claire knew that she'd need to think about differentiated language instruction and assessment for her 11 students at the early advanced and advanced levels of English language proficiency.

PLANNING FOR CONTENT AND LANGUAGE INSTRUCTION IN A DUAL LANGUAGE SETTING

In one of their earliest meetings, Eduardo provided Claire with a copy of *Dual Language Essentials for Teachers and Administrators* (Freeman, Freeman, & Mercuri, 2005). He specifically shared the section on preview/view/review (pp. 99–103), which he explained would guide their collaboration. In Freeman and colleagues' words:

> Preview/view/review allows teachers to make the second language more comprehensible by giving an introduction, or preview, in the students' first languages, then teaching the content in the second language using a number of techniques to make the input comprehensible, and finally reviewing after the lesson in the students' first language. (p. 100)

Eduardo also directed Claire's attention to the explanation of how this approach works in the dual immersion context: "As long as the languages alternate, what is presented in a student's first language provides preview and review for the lessons presented in the second language when the curriculum is organized around themes" (p. 101). He made sure to emphasize from the beginning that their planning would not result in a direct translation of lessons, but rather, a conscientious focus that concepts from week to week were complementary, providing footholds in the students' home language in order to more efficiently acquire language and content in their second language.

Eduardo also highlighted the importance of being aware of the students' stronger languages (for particular language domains or situations) as well as their number of years in the program. Students who had attended the school since kindergarten were more likely to be approaching a relatively high level of competence in both languages; however, late-entry students might demonstrate a lower level of proficiency requiring more scaffolding. For those students, it was even more vital to understand which language is providing the preview and review versus the language of viewing, which would be cognitively more challenging and draining.

Consider this . . .

Consider how you can make time to engage in dialogue with your colleagues in order to discuss the vision of the education you would like your students to experience. Consider how you might ensure that your daily practice aligns with this vision.

Once Claire had a chance to process the assumptions behind preview/view/review, she and Eduardo began to discuss how they would effectively support the students they would coteach throughout the year, including how they would set up their rooms. During brainstorming, Eduardo was impressed with some of Claire's ideas and knew that they would also be beneficial in his classroom. They drafted up ideas for many of the physical aspects of their classrooms to mirror each other—with the exception of the difference in language. As a result, both of their classrooms had chart paper tablets to record daily objectives, math news bulletin boards, and word walls on which to post academic words and phrases throughout each unit.

Claire's and Eduardo's Teaching Practice

In conversations prior to the start of the year, Claire and Eduardo discussed how their teaching philosophies for their classrooms served the purpose of having a common vision for the type of math environment they wanted to create for their students as a teaching team. Eduardo knew that Claire had pursued a mathematics teaching specialization as part of her teacher preparation program, and that she had successfully passed middle level mathematics licensure tests. His own specialization was in Spanish language arts, so he appreciated the expertise in math that Claire brought to her students.

Eduardo recognized that as Claire prepared for her first year of teaching, she would need to have some time and space to develop who she wanted to be as a teacher, rather than having the way it had to be dictated to her. As a result, he provided some of his thoughts and answered her questions but also wanted to be careful to allow her to feel valued as a professional capable of making decisions right from her first year. He made a conscientious decision to provide suggestions when necessary but to also celebrate what she was doing right in order to support her in developing her effectiveness as a teacher.

From time to time, Eduardo was able to observe Claire teach and provide feedback as her mentor. As Eduardo watched the visual elements of Claire's classroom evolve over the year, it was easy to see what Claire valued as a teacher. They continued to talk about the intentional decisions they were making to set up their classroom community, and often, when

they discovered the positive impact of an aspect in one classroom, they considered whether or not to layer it in as a mirrored aspect in the other classroom.

For example, Claire liked the way Eduardo was able to emphasize content objectives orally and in written form. Adopting a practice she had seen in Eduardo's room, Claire made sure to post objectives for each content area lesson throughout the day. Across the back wall were two large chart paper tablets on hangers underneath signs that stated a grade level (one each for fifth and sixth grade, depending on the week), and the day's objectives written on these charts by students as part of each lesson's opening routine.

Other aspects of Claire's classroom mirrored Eduardo's use of visuals and his large classroom library with featured books in the target language tied to content areas. The back corner had Claire's mathematical library with a changing array of texts from picture books to young adult books that somehow involved mathematical ideas. A few three-dimensional math puzzles and math games were in the library as well. Signs at the front of Claire's room, close to the entry, stated, "Use your minds. Use your manners. Use your morals." Above the bookshelf was a math news bulletin board for students to post examples of math in the real world, such as magazine and news articles. To the right of the books and bulletin board was a "Terms & Symbols" mathematics word wall.

When planning units, Claire and Eduardo collaborated in order to determine the key academic mathematical terms, expressions, and symbols, updating their word walls to represent the content instruction from both weeks. Eduardo explained that Claire would want to include the concepts he introduced in Spanish (and vice versa) so that both teachers would be able to build on students' previous understandings of target vocabulary and content taught in each language. The explicit link to vocabulary from the opposite weeks would aid in students' transfer of skills and allow them to attach a label to terms in the target language of the week. The two teachers provided common visuals between the two classrooms to accompany the text on the word wall. In this way, they provided one more element to help students make connections to the terms used in both classes without sacrificing the sense of immersion.

To Eduardo, Claire's emphasis on critical thinking and a student-centered environment was apparent with the way that she structured her class. For example, she started each day with a math journal, by having students respond to a prompt on the Smart Board. She circulated to observe how students were approaching the prompts. He admired her facility with feedback, which was in the form of thinking prompts, gentle reminders to help students extend their thinking and make connections

between what they already knew and the current prompt. For example, she sometimes highlighted a word or phrase in the prompt and said, "Think about what that might mean."

She also encouraged students to validate their thought processes rather than give "right" answers alone. She stated, "Remember your math journal is a place for your ideas. It is better to have an idea than no answer." Math journals in Claire's classroom were a means to assess prior knowledge on a topic or to review a concept from previous lessons. Student responses prompted Claire to adjust her lesson to be responsive to students' demonstrated depth in understanding or to clarify any misconceptions or misunderstandings.

When Claire shifted to whole-class discussion of the prompt, she involved the students rather than simply showing how to approach it on the board. Students in Claire's class were used to explaining their thought processes and to having these processes documented on the board, either by Claire or by themselves. Claire encouraged and welcomed aspects that were challenging for students. She often brought up areas that stumped students while she was circulating and observing. This strategy prompted students to think through mathematical prompts as a group and encouraged supportive, problem-solving dialogue in a community of learners.

She also pushed students to approach prompts through multiple avenues. Eduardo thought that her actions embodied the Chinese proverb, "If you give a man a fish you feed him for a day. If you teach a man to fish you feed him for a lifetime." Claire did much of this intuitively without necessarily realizing how it was a strong layer of support for her ELLs. When debriefing, Eduardo was able to highlight the advantages of her regular classroom practice. In turn, she was able to intentionally focus on consistently providing this support for her students.

Eduardo appreciated the fact that during math instruction Claire frequently posed questions starting with, "What did we discover when . . . ?" Eduardo thought that her students were learning to recognize the value in paying attention and noticing ideas related to mathematics.

Claire's emphasis in her teacher preparation program was in math, not English as a second language (ESL). Nonetheless, because Claire had been an international ESL teacher, she understood a lot about how to support language learners. Eduardo had noted that her pacing was appropriate, and she was intentional about her use of academic language supported with visuals and manipulatives.

Claire was also good at reviewing the language in their adopted math text for words and phrases that might be unclear to her ELLs. Eduardo liked the way she was constantly monitoring the language

used in word problems in the text: "In this problem about hiking, I see the phrase 'mile markers.' Who here has been on a hike? What kind of markers are meant here? Are these the markers that we use in school to draw?" Eduardo also saw that Claire typically spoke very clearly, enunciating her words. She usually avoided confusing idioms and casual language in her teaching. She was an excellent monitor of her own language use, as this exchange illustrates:

Consider this . . .

Take a look at your math book. What other words, such as *markers*, have you found in story problems that have multiple meanings?

Claire: So, if you've ever been bakin,' measuring ingredients—

Lilli: Oh, I love that! And I love sausage too—

Claire (laughing): I should have said that more carefully, Lilli! I meant, *baking*, as in cooking something in an oven. For example, I was *baking* a cake, after I mixed ingredients that I had measured carefully. Not *bacon*, or sausage—but I love that, too!

While Claire appreciated having Eduardo provide a background for the dual immersion model, he appreciated seeing her quality teaching for language learners in the specific content area of mathematics. Though Claire was deepening her understanding on a daily basis, Eduardo knew that she identified meeting the needs of language learners as an area in which she would like to improve. By collaborating with colleagues, through discussions and regular team meetings, and in close observations of her students and deep reflection, Claire tried over the year to adapt instruction to better align with demonstrated student needs. Both Claire and Eduardo felt they were a good team in planning mathematics instruction and their other content areas.

UNIT THEME: THE POSSIBILITIES OF GEOMETRIC SOLIDS

At the start of the school year, Eduardo had been instrumental in providing a strong framework for each unit they cotaught, allowing Claire to familiarize herself with other teaching routines and responsibilities. As the year progressed, the responsibility shifted to a more collaborative coplanning, and as they approached the end of the year, Claire asked to take the lead on planning a unit, so she could apply all that she had been learning from the modeling and collaborative planning earlier in the year to the geometry unit they

would be teaching soon. She knew that the first step was to closely examine grade level standards to gain a clear understanding of essential concepts and understandings that she would like her students to learn during the unit, both in content area concepts and in language development in English.

Adopted Textbook

The adopted textbook was a logical starting point. For grades 6–8, Claire's school was using a Holt textbook series; the text for sixth grade, *Holt Mathematics: Course 1* (Bennett et al., 2007), was available in both English and Spanish. This series included helpful tutorials as well as games and tips for parents online, although, unfortunately, not all of these supports were available in Spanish. Even though the school's immersion model discouraged direct translation in favor of total immersion in the language of instruction for any given week, Claire knew that many parents really appreciated the English text's provision, in each chapter, of key vocabulary in both English and Spanish.

As she scanned the textbook, Claire saw that Chapter 10 focused on solving problems involving area, including surface area of prisms, pyramids, and cylinders; it highlighted finding the volume of prisms and cylinders. This chapter would be of great help in meeting the sixth grade math standards for geometry that Claire had identified, although she would need to provide a stronger focus on helping students find the surface area of spheres and cones. Claire appreciated the chapter's links to real-life applications for geometric measurement of surface area and volume, which included geometric shapes in architecture and featured careers in landscape design. The chapter started with calculating volumes of solids before exploring surface area calculation, but she and Eduardo planned to teach surface area first and volume second, as students worked on first covering, and then filling, their piñata shapes. So, they scoured to find an appropriate example of surface area in their text.

Supplementary Resources

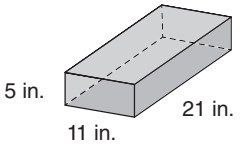
In addition to the adopted textbook, Claire also wanted to consider how she could utilize supplementary resources to foster an even greater sense of connections to geometry in real life.

As they brainstormed, Claire and Eduardo realized that a traditional event, their school's Día de los Niños (Kids' Day) celebration, could reinforce their introductory study of geometric forms. If students identified geometric forms within the school's piñatas for the celebration, and then built and filled the piñatas, they could actively engage in direct measurement of, and

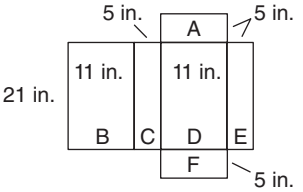
Excerpt From *Holt Mathematics: Course 1*

10-9 Surface Area

Katie made a toy for her cat to scratch by attaching carpet to the faces of a wooden box. The amount of carpet needed to cover the box is equal to the surface area of the box.



The **surface area** of a three-dimensional figure is the sum of the areas of its surfaces. To help you see all the surfaces of a three-dimensional figure, you can use a *net*. A **net** is the pattern made when the surface of a three-dimensional figure is laid out flat showing each face of the figure.

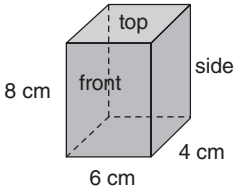


Example 1 Finding the Surface Area of a Prism

Find the surface area *S* of each prism.

A. Method 1: Use a net.
Draw a net to help you see each face of the prism. Use the formula $A = lw$ to find the area of each face. Add the areas of each face.

B. Method 2: Use a three-dimensional drawing.
Find the area of the front, top, and side, and multiply each by 2 to include the opposite faces. Add the areas of the faces.



Source: Bennett et al., 2007, p. 582.

mathematical calculation of, surface area and volume of different geometric solids. A few parents who had offered to help with celebration planning could be invited to the math classroom on the days when students were working with solids and building forms into piñatas. While the students' piñatas would be simple—merely wrapped cardboard forms attached together and filled with candy—Eduardo eagerly began a search for Spanish language YouTube videos that documented traditional piñata making and looked for ways for his students to link mathematical reasoning to what he selected as video clips for the Spanish math block.

Claire knew that she and Eduardo would want to scaffold more connections between geometry and real-life applications throughout the unit. While exploring sites, Claire found a blog of fifth graders talking about real-world applications for geometry: Real Life Applications for Geometry (http://geometry5th.wordpress.com/2010/05). Even though it appeared that it was no longer updated, Claire thought about how it would still have potential benefits for her students, especially as a model for a blog that they would create. She decided to use a section of this WordPress blog post as a supplementary resource for her students. They would read it, discuss the content, note any need for editing, and then create their own math blog posts, linked to the school's website.

Supplementary Text: Student-Produced Geometry Blog Posts

WordPress Blog Page Posts

Chase

May 17, 2010

Knowing the volume of a prism would be helpful in real life for someone at a packing company needing to know how much packing foam to put in a package. It would also be helpful for a pizza company knowing how big to make it so that a pizza will fit, or a person who wanted to fill a fish tank.

Tags: Chase

Posted in Uncategorized | 5 Comments >>

Luke

May 17, 2010

Imagine this, you are a brilliant painter of Picasso potential. You are painting a house and you need to know how much paint you will need. First, you need to find the surface area of the prism shaped house. You will need to know how to find that surface area. Then once you find that, you will know just how much paint you will need to complete the house. Another example is if you are a roof constructor and you have to put shingles on a house. You need to find the surface area of the roof so that you know how many shingles you need.

Tags: Luke

Posted in Uncategorized | 4 Comments >>

Lauren

May 17, 2010

Suppose a water tank in the shape of a right circular cylinder is thirty feet long and eight feet in diameter. How much sheet metal was used in its construction? This is an example of a real life application for finding the surface area of a cylinder. Anyone from a plumber to a painter would need to know the formula. A painter would need the formula so that they can decide how much paint they need to get if they are painting an object in the shape of a cylinder.

Also, someone who makes toys would need to know how much material is needed if they were making an object in the shape of a cylinder. Surface area can also be used if you are trying to find out how much wrapping paper to wrap a gift. Surface area is used for many things from gifts to plumbing.

Tags: Lauren

Posted in Uncategorized | 5 Comments >>

Source: http://geometry5th.wordpress.com/2010/05/

As Claire and Eduardo looked at their math text (they had versions in English and Spanish) and at their proposed supplemental text (the student-produced WordPress blog), they realized that there was a range of academic language across those materials for Claire's students to learn in the first English week of their unit (see Figure 2.2).

Figure 2.2 Range of Academic Language Students Would Encounter in Claire's English Language Text and WordPress Blog Supplementary Text

Text Type	*Discourse Level*	*Sentence Level*	*Word/Expression Level*
Grade-Level Mathematics Text	Definitions (of surface area)	Declarative sentences: The *surface area* of a sphere is calculated by. . . . Imperative sentences: Draw a *net.* Use the formula _____ to find the surface area of a _____.	surface area three-dimensional figure net face base formula side diameter circumference prism cylinder pyramid cone sphere
WordPress Blog Post	Explanations of processes (used in calculation)	Temporal adverbs: First, _____. Then, once you find that, _____. Conditional sentences: If you wanted to paint a house, you would need to know the surface area of the house. Imperative sentences: Imagine that. . . . Suppose you. . . .	gerunds ("knowing the volume of a prism would be important because . . .") packing foam shingles constructor sheet metal plumbing

CONTENT AND LANGUAGE STANDARDS TO GUIDE INSTRUCTION

Content Standards

Looking at the sixth grade Common Core State Standards for geometry adopted by her state, Claire identified the standards in Figure 2.3 as being relevant for the unit.

Claire knew that it was also essential to understand the geometry standards from other grade levels. Because she also taught fifth grade, she

Figure 2.3 Oregon Common Core State Standards, Mathematics

Geometry 6.G	H. Solve real-world and mathematical problems involving area, surface area, and volume.
6.G.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
6.G.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
6.G.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
6.G.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Source: Oregon Department of Education, 2010b, p. 8

was familiar with those standards. She considered the background that students might already have, and that she might need to assess their knowledge to determine whether they needed additional background lessons prior to moving into instruction related to sixth grade standards. She also viewed the seventh grade Common Core mathematics standards to understand the foundation that current instruction will provide in order to set students up for success the following year.

When considering the three grade levels side by side, she noticed real-life mathematical applications; however, each was related to a different mathematical focus. She also noticed the other two grade levels in the OR CCSS having two clusters rather than one (see Figure 2.4).

According to the standards, fifth grade students would have gained a foundation in two-dimensional figures, which would support their understanding of sixth grade expectations, specifically with area. Claire thought about how the concept of three-dimensional figures related to surface area and volume might be relatively new to her sixth grade students and would require more manipulatives and real-world applications in order for students to grasp the concepts. Additionally, the fifth grade focus on graphing

Figure 2.4 Oregon Common Core State Standards, Mathematics

Fifth Grade	*Sixth Grade*	*Seventh Grade*
J. Graph points on the coordinate plane to solve real-world and mathematical problems.	H. Solve real-world and mathematical problems involving area, surface area, and volume.	F. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
K. Classify two-dimensional figures into categories based on their properties.		E. Draw, construct, and describe geometrical figures and describe the relationship between them.

Sources: Oregon Department of Education, 2010a, p. 3; 2010b, p. 4; 2010c, p. 4.

points on coordinate planes would support the sixth graders' understanding of concepts outlined in 6.G.3., involving drawing polygons in the coordinate plane. Once again, Claire knew it would be advantageous to assess students' backgrounds on coordinate planes prior to the first lessons in her unit in order to determine her starting point.

With regard to seventh grade standards, Claire saw that students would have opportunities to further develop their understanding of area, surface area, and volume. For this reason, she carefully examined the cluster descriptors of the two grade levels side by side in order to focus on what to emphasize within the general umbrella of area, surface area, and volume for her sixth graders. One example of a difference was that the sixth grade worked on polyhedrons (or geometric solids with faces), while in seventh grade, students included the surface area and volume of spheres and cones. Claire reminded herself that it would be necessary to frequently refer back to the standards while continuing to plan instruction and to share them with Eduardo.

Language Standards

After selecting the content standards, Claire thought about which Oregon English Language Proficiency (ELP) standards would naturally complement the content and align it to students' needs. For her early advanced and advanced ELLs, she decided to focus on the following language functions: comparing, contrasting, and explaining, as illustrated, along with examples of corresponding grammatical forms, in Figure 2.5.

Figure 2.5 Oregon English Language Proficiency Standards, Language Functions

Language Function	*Early Advanced*	*Advanced*	*Grammatical Forms*
Comparing	Varied sentence structures with specific comparative adjectives and phrases	Complex sentence structure with specific comparative language	Adjectives and Conjunctions
Contrasting	Subject/verb/adjective, *both* subject/verb, *but*	Approximately used idiomatic phrases and contrasting words (e.g., *whereas*, and *in contrast*)	Comparative Adjectives
Explaining	Get across important points using declarative, compound and complex sentences, regular and irregular verb forms. Complex: *"As I came home, I stopped at the store."* Compound: *"The children who came in early had refreshments, but those who came late had none."*	Get across which point he/she feels is most important using regular and irregular verb forms, adverbs of manner and compound-complex sentences. Adverbs of manner: *The children who sang loudly got a cookie, but those who didn't sing had none.*	Verb Forms—Indicative verb *(makes a statement of fact)*. Declarative Sentences, Complex Sentences, Adverbs of Manner: Students learn to develop and use explanations using appropriate verb forms, declarative and complex sentences and adverbs of manner.

Source: Language Functions and Forms, n.d., pp. 7, 9.

Academic Language

Claire and Eduardo knew that their short unit on geometric solids would include challenging new vocabulary and grammar for their sixth graders. Together they considered the range of English and Spanish academic language in the texts and in the activities they were considering (see Figure 2.6).

After creating a clear picture of the standards and materials that would guide their instruction, Claire and Eduardo formulated the language and content targets for the unit.

Language and Content Targets

Wanting above all to get their students excited about the possibilities for geometric forms, Claire and Eduardo decided that a short unit, with the first four days in English and the second four days in Spanish, would

Figure 2.6 Range of Academic Language in Unit on Geometric Solids—English and Spanish

Activity	*Discourse Level*	*Sentence Level*	*Word/Expression Level*
Measuring surface area of real geometric solids (wrapping activity)	Explanation of measuring processes	Adverbs: First, we_____; then, we_____. Finally, our calculations showed that _____. Comparative language: The easier way to calculate the surface area of a sphere is_____.	three-dimensional net polyhedron cube cone cylinder sphere, half-sphere pyramid surface area face side base
Surface area formulas for geometric solids Volume formulas for geometric solids	Word problems in text		surface area volume formula quantity
Piñata activity (filling with candy)	Directions		sculpture piñata cubic

have calculation of surface area and volume of six geometric solids as the overall content target. Claire thought that the language functions from the ELP standards might be useful in formulating the language target. Here is what the pair decided on for the unit's targets:

Content Target for Students:	Students will solve and explain mathematical problems involving surface area and volume of cubes, rectangular prisms, cylinders, cones, spheres, and half-spheres.
Language Target for Students:	Students will compare and contrast methods for calculating surface area and volume in a variety of geometric solids, including their piñatas. Students will explain why people in their community would need to calculate surface area of a variety of geometric solids.

The unit test would be in English, although daily homework during Spanish week would be in Spanish. As usual, formative assessments would be initiated by each teacher in the language of instruction used at the time. Claire would start the unit with explorations of surface area in geometry, having students explore calculations of surface area by wrapping various geometric solids that would be used either as piñata parts or as manipulatives in English and Spanish math classrooms. She would also start their real-world connections to geometry blog posts in English.

Eduardo would continue with explorations and calculations of volume of the hollow solids that students had created. Students would create blog posts in Spanish with pictures of the created piñatas. The last day of Eduardo's week would be spent in celebrating Día de los Niños with the whole school, and when Claire met with students again the following Monday and Tuesday, she would review concepts and administer the textbook unit test that covered geometric solids, surface area, and volume.

LINGUISTIC AND CULTURAL RESOURCES

When Claire and Eduardo first met to plan their initial geometry unit for sixth grade, Claire knew that she had some planning considerations unique to their setting. For example, she wanted to make sure her students saw possibilities for geometry across languages and cultures in art and in community architecture in different countries. She didn't want geometry to just be "memorizing formulas around geometric solids. . . . This intro unit needs to be about possibilities." She wanted to tap into students' and families' funds of knowledge about geometric forms in a concrete way, and for them to recognize how surface area and volume calculations figured into their families' lives. She wanted to better assess and document her students' oral and written use of academic math language. And finally, both Claire and Eduardo wanted to tie mathematical thinking into their students' and any interested parents' planning and preparation for their annual April festival day of celebration of children, or Día de los Niños.

INSTRUCTIONAL SUPPORTS

Word Wall and Math News Board

Both teachers had ongoing word walls in their rooms, where they added new math terms as the units progressed. In addition, students watched for examples of math in the real world to bring in and post on the math news boards.

Extra Articles/Readings/Math Library

In addition to a permanent math library in each classroom, the teachers also accessed resources such as the public library and the Internet to include additional books and texts specifically related to the current content.

Realia

Claire and Eduardo know it is essential for students to be able to visualize the math concepts. For this reason, they considered which hands-on manipulatives they could include with each unit, such as the range of geometric shapes included in this unit.

Math Journals

The students' daily routine started with an entry in a math journal to prompt their thinking about new content and/or to refocus their attention on key concepts from the previous day that would serve as a foundation for the day's lesson.

Encouraging Student Interaction

The teachers balanced individual reflection and thinking with peer interaction. For example, after completing their math journals, students were often asked to turn and talk to partners close by prior to discussing as a class.

Technology

Both teachers utilized technology available at the school to highlight the math content, such as Smart Board interactive options, student blogs, and YouTube videos.

PLANNING FOR INSTRUCTION AND ASSESSMENT

After teaching nearly a year referencing Oregon state standards and with an eye to multiple forms of data collection with Eduardo's support, Claire had some good ideas for assessing students' knowledge of geometry over the unit. She had given part of the unit pretest that covered surface area and volume to students, noting that while most could name geometric solids in English and could calculate surface area of a one-dimensional rectangular

figure and a solid quadrilateral using a "net" approach, most had little understanding of calculating surface area of curved solids or in figuring volume of geometric solids. Other content data she would collect to assess progress would be calculations from daily math journals, homework problems from the text and that she made up for them, and an end-of-unit test.

Claire was less certain about how to assess students' oral and written use of mathematical and academic language. While she paid a great deal of attention to students' oral explanations as part of her teaching, her collection of oral language data—and her reflection about it—was "all in my head." To improve her documentation in this area, for this unit, she planned to devise a quick checklist for assessing the students' oral explanations as pairs demonstrated how to calculate surface area. When she asked Eduardo if he had any suggestions, he showed her a sample rating scale from a book that the school's instructional coach had just given him: *Common Language Assessment for English Learners* (Gottlieb, 2012).

Claire studied the sample oral rating scale and adapted it, based on the language demands of her sixth grade geometry unit and Oregon ELP standards (Oregon Department of Education, n.d). She made eleven copies of the protocol, one for each ELL, and put them on a clipboard. As she circulated among students working in pairs, Claire would jot down some notes about the students' use of explanatory language and then report her discoveries to the whole class. All her students were used to anecdotal record-keeping, so Claire knew they wouldn't be bothered by her note-taking as they worked. Figure 2.7 shows the rating scale used during anecdotal record keeping.

Eduardo was interested in knowing how the rubric worked with the students, since it was a new resource for both of them. Because of Claire's positive feedback, they decided that he should create a similar one in Spanish to use when Spanish language learners reported on volume calculation with the same geometric solids during the Spanish week. Therefore, the two would have a common documentation form that they were both using, which would facilitate discussion with data teams when considering students' oral academic usage. This would allow them to consider students' strengths and areas of growth.

Finally, Claire and Eduardo decided that they would utilize the student-created blog posts to assess students' ability to explain their learning to others, in this case their parents and the overall school community. In English week, Claire decided to use the fifth grade blog she found to show students as a starting point of the possibilities. She wanted to build on the students' curiosity related to geometry in everyday objects to support them in gathering ideas and drafting their geometry blog posts. Eduardo was glad to layer in a different type of student blog post in Spanish to highlight the process of creating their own piñatas and presenting them to

Figure 2.7 Sixth Grade Speaking Protocol for Early Advanced and Advanced ELLs

Purpose: Rate students based on evidence from oral pair work and pair's oral report to class on (1) explaining challenges and processes in surface area calculation activity with geometric solids, and (2) use of comparing and contrasting language in discussion of direct measurement versus mathematical calculation of various lengths in geometric solids as part of surface area calculation

Date:_____ Student:_____ Language Level:_____

Meeting Language Objectives:	*Consistently (if checked, include evidence from student's speech)*	*More Often Than Not (if checked, include evidence from student's speech)*	*Infrequently*
Early Advanced: Explaining Use of regular and irregular verb forms in past tense, with declarative, complex, and compound sentences*: We measured the sides of all the faces. As we measured, we wrote down the lengths.*			
Advanced: Explaining Use of regular and irregular verb forms, especially past tense, adverbs of manner*: First, we measured the length around the middle of the sphere, in order to find the circumference.*			
Early Advanced: Comparing/ Contrasting Use of comparative adjectives, varied sentence structures: *Finding the length of each side of the pyramid was easy, but finding the area of each face was harder.*			
Advanced: Comparing/Contrasting Use of language involving comparative adjectives with complex sentence structure and specific comparative language and contrasting words: *For a sphere, using the tape measure to directly measure the radius was hard, because we couldn't cut the sphere in half. In contrast, using the formula to calculate the radius was a lot easier.*			

Source: Oregon Department of Education, n.d, pp. 7, 9, 10, 11; Gottlieb, 2012, p. 60, Figure 3.6.

the younger students. Because students would be writing posts in both languages, they would be able to share their language and content learning in the unit with both native English- and Spanish-speaking families. A sample blog post is shown in Figure 2.8.

All of the teachers talked about how they would intentionally focus on the Oregon ELP standards in their lessons and stay in close communication to consider layers of support needed as the unit unfolded. Just as Claire had done with her speaking rubric, they planned to create a blog post for all students with special attention to ELLs with early advanced and advanced language proficiency levels for writing.

Priming the Pump: Generating Interest in Geometric Solids

Two weeks before beginning their unit on geometric solids, Claire and Eduardo quietly began "priming the learning pump" for a new unit by

Figure 2.8 A Student Spanish Blog Post Written Collaboratively

TUESDAY, MAY 1, 2012

Proyecto de Matemáticas: Piñatas para el Día de los Niños

Nosotros preparamos un proyecto de hacer piñatas para el Día de los Niños. Habían muchas figuras geométricas diferentes. Primero, en la semana de inglés trabajamos en parejas para encontrar el área de la superficie con diferentes herramientas como cintas de medir. Usamos la hoja de fórmulas para encontrar el área de la superficie de la figura que nos tocó. Algunas figuras eran más complicadas que otras. Las esferas nos dieron más trabajo para encontrar el area de la superficie. Los cubos fueron los más fáciles de todas las figuras.

La razón por la cual estabámos midiendo el área de la superficie era para saber cuánto papel necesitábamos para cubrir las piñatas. En la semana de español, aprendimos como calcular el volumen de las figuras para saber cuántos dulces necesitábamos para llenar las piñatas. En una asamblea presentamos las piñatas a todos los niños de la escuela. Cuando los niños vieron las piñatas se emocionaron.

Posted by Mrs. V. at 2:40 PM

tailoring their physical classrooms and their student math journal prompts into funds of knowledge about geometry. During these two weeks, the sixth graders were taking state-mandated mathematics tests, so most of their instruction involved review activities in small groups as individual students left class to take benchmark tests in the computer lab. Claire and Eduardo intentionally thought about how they could make the content of those weeks meaningful within the scope of units during what could otherwise be a disconnected frame of time with students in and out of the classroom.

Claire took advantage of the review week to prime the pump with some of her early advanced ELLs by introducing three students to an article she had found in *Scholastic* magazine titled "Sandy Solids" (Wise, 2009). This material was written for an easier reading level than grade 6, but it featured content of interest and grade-level academic language for her students. The short article reported the use of volume calculation among sand castle artists, and it included a good discussion of how the sand builders calculated how much sand they would need for their huge sand creations. She invited the students to put the article on the math news board the next day and to give the class a brief summary of it. As they reported to the whole class, Claire identified these three developing ELLs to their peers as "the volume of solids experts" and told the class that even though they wouldn't be covering volume calculation in English math, the topic would come up soon in their Spanish class in their geometry unit.

Meanwhile Claire was busy locating and displaying other texts related to geometry. Since she had access to a wealth of databases through her former university's library system, finding relevant articles from a variety of math and science magazines was merely a matter of time spent on her computer. She was careful to find two more articles of different reading levels but of high interest to her students, and she identified several YouTube videos (featured in the section below, Kicking Off the Unit) about the work of modern artists from different Spanish-speaking countries around the world; their work focused on use of geometric forms in two and three dimensions. Eduardo engaged in similar activities with students in his Spanish review week.

Claire also found a number of great trade books in English about geometry through her public library system. (She had to order many from around the state, but fortunately her rural library was part of a much wider library system.) Claire featured these

Consider this . . .

How do you prime the pump and generate interest in an upcoming unit? Where can you find articles and video resources to extend your content instruction for students with different reading and listening proficiency levels?

books prominently in her classroom library and was pleased when several students checked some of the chapter books out for reading credit in their language arts class.

Claire made sure to prominently display on the whiteboard tray at the front of the room two illustrated picture books and one chapter book that featured illustrations by artists who celebrate geometric shapes in their work.

Consider this . . .

How would you build your math library? Where can you find current resources and how might your school budget for them?

Since she knew she would need a lot of lightweight geometric forms for students to handle, Claire called one of her parent volunteers for help. Over the next few days, the parent worked with other parents and friends to find a range of geometric shapes that students could measure, wrap, and save (from cardboard boxes of different sizes to coffee cans and oatmeal boxes for cylinders, hollow cardboard cones used for hat making, and florists' foam spheres, cubes, and pyramid shapes).

Finally, Claire began shaping her prompts for each day's math journal to her students' geometric funds of knowledge and carefully reading their responses from their math journals to gauge their background knowledge and connections to geometry. On the day she prompted, "When do members of our community calculate the surface area of different shapes?" she gained a great deal of information from students that indicated she had learners who had a lot of knowledge, if not the academic vocabulary, about how geometric calculation informed different work in their community.

One boy wrote, "My sister made a tile table in her high school construction class. She had to know the surface of the table, and how big the tiles were, to see how many could fit in the design she wanted to make of the tiles." Other students wrote about estimating amounts of materials for roofing the clinic being built next to the school, filling a flatbed with hay bales, and knowing how much frosting to put on an entire wedding cake. One student's "sowing cloths" response puzzled Claire, until she asked the student to "please tell me more about this—I am interested!" The girl responded that her aunt made her a lot of clothes, and when she prepared to sew from a pattern, she needed to know how much fabric she needed and how to alter her pattern pieces to cover her niece's "body, arms, legs"—a clear link to surface area calculation. In fact, it was one student's math journal discussion of

Consider this . . .

Claire makes daily use of a math journal. She feels that students' entries in their journals give her insight into their math thinking and language development. How might you incorporate a math journal into your instruction? What kind of writing prompts would you include?

knowing how much tissue paper was needed to wrap a present that led Claire to think about how the class might create its own geometric solids for use as manipulatives and visual aids in English and Spanish classrooms.

KICKING OFF THE UNIT: A LESSON ON WRAPPING GEOMETRIC SOLIDS

Claire looked around her room and drew a deep breath. She felt she was trying to fit a lot of activity into the day's sixth grade math block in this first lesson of the unit. She had plenty of materials for the wrapping activity on a side table: a variety of geometric forms ready to be handled, measuring tools (rulers, string, flexible tape measures, scissors), sheets of brightly colored tissue paper, tape, and glue. She had students' speaking rubrics on her clipboard, ready to record evidence of their language use during the activity; her word wall had key terms (*pi, circumference, diameter, radius, face, polyhedron, rectangular prism*); geometric formula sheets were available; a tourist's YouTube video clip featuring a current installation of Rafael Barrios's sculptures in New York City was cued up on the smart board; she had a parent volunteer to help her students as they worked in pairs. She herself wasn't going to introduce any surface area formulas for solids or have her students do any surface area word problems first; instead, she hoped today's hands-on experience would set them up for math problems in surface area in the coming days.

Claire's students began to come in and settle down to write for a few minutes in their math journals. Claire's prompt was "Where do you see polyhedrons in our community?" As they wrote, Claire asked one of her weekly helpers to write the day's objectives on the sixth grade chart for everybody to see:

Content Objectives

1. Working in pairs, we will calculate the surface area of different geometric solids.
2. Based on our calculations, we will wrap geometric solids using the least possible amount of tissue paper.

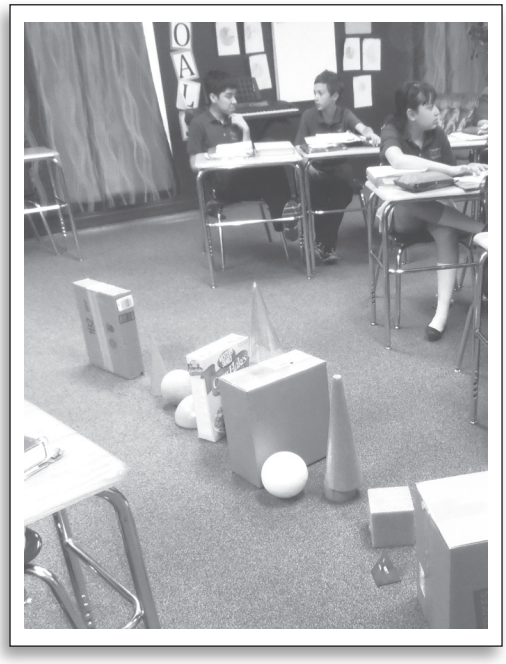
Language Objectives

1. We will *compare* geometric forms and *describe characteristics* that are similar or different.
2. We will *define key vocabulary* words that we need: *polyhedron, cube, cylinder, pyramid, cone, rectangular prism, sphere, two-dimensional* and *three-dimensional.*

Once the day's objectives had been read aloud by the weekly helper, Claire addressed the class. "We've been circling around—get it?—a new unit in geometry. What is geometry about?" Students offered some responses "using pi," "figuring the volume," "building models," and "shapes." Claire smiled and said, "Yes, all those—and shapes especially. Now, in these picture books on the chalk tray, we see artists' work where shapes are very important. See this book with the strong woodcut-type illustrations? Those shapes are flat; they are two-dimensional. Now, what about shapes in three dimensions, like the shapes that pop out at us in a 3-D movie?"

Claire asked students to name some three-dimensional shapes and drew their attention to names for those geometric solids on her word wall. "To start off today, we're going to watch a really short video clip that features the sculptures of Rafael Barrios. He is a Venezuelan artist, and his work includes huge three-dimensional shapes. Some of his art is being shown outside in New York City right now. As you watch, think about all the shapes you see. Next week in Spanish math you'll see an interview in Spanish with him about his work, but today just take a look at those sculptures, with all their different faces, and imagine what it would take to cover them up to transport them here" (see hethandjed, 2012).

Figure 2.9 Students Think About How to Calculate Surface Area



Once the students watched the video, Claire pointed to a bulky garbage bag in the middle of the floor. "Now, we've just seen some huge shapes. What do you think I have in here?" The students' interest was piqued.

One by one, she pulled out various geometric solids and asked kids to name them (see Figure 2.9). "Now, why might anybody be interested in the surface area of shapes?" Students offered some responses: "to cover up a sculpture," "to cover a round pillow or a square pillow," "to wrap a present," "to make the skin of an airplane."

Claire acknowledged students' responses, and then explained: "Well, today we have a reason to find the surface area of different geometric solids. We want to know the least amount of

tissue paper it would take to wrap up these solids here, because we can't waste any paper. And we'll use our covered solids for math class. Some of the hollow solids we will use to make piñata parts for Día de Los Niños! And by working really hard on volume next week, we'll know how much candy to put into each."

Claire and the parent volunteer quickly modeled how they might find the surface area of a cube-shaped box, asking for student input along the way. They used rulers to measure each side and emphasized that they were figuring out area, so their results would be in square units. Claire explained that students would work in pairs to calculate the surface area of various solids. They could use any of the measuring tools at their disposal, as well as refer to the Official Formula Sheet and Conversion Table for Grades 6-8 developed to math test specifications by Oregon's Department of Education (Oregon Department of Education, 2010/2011).

Once students had calculated the least amount of tissue paper needed, they would show their calculations to Claire and then cut appropriate sections of tissue paper to cover their solids.

Before handing out a solid to each pair, Claire asked, "Can anyone guess which shapes might be hardest for us to calculate their surface area? Why?" Sofia and Maria spoke at the same time: "The curvy ones!" "The round ones, because the sides aren't straight so you can't just multiply!" Claire smiled and said, "You might be on to something there. I will be circulating to provide scaffolding as you work . . . we'll be on the lookout for challenges in direct measurement."

Claire called students up in pairs and handed each a solid. Much of the time, her students choose their own partners or groups. Before this lesson, however, she had carefully paired up the students so that they were in each other's zones of proximal development in English language proficiency. For this activity, she had made sure that each ELL was paired with a student whose English proficiency was slightly higher than his or her own.

Consider this . . .

Consider what you know about Lev Vygotky's *zone of proximal development.* This term refers to the distance between what a learner—in this case, an ELL—can do with assistance, and what the learner can do independently. Claire knew many of her ELLs were good at mathematical reasoning, but they didn't always have the academic language to explain their math reasoning processes in English. Claire grouped students so each early advanced or advanced ELL was paired with a highly proficient English speaker, because she wanted the students to focus on explaining their math reasoning. Do you think her grouping strategy was sound? When do you try to put students with other students within their zones of proximal development?

Students began to work. Claire circulated around the room, noticing that those pairs who had been given rectangular prisms were moving quickly to calculate surface area.

Julio: Yeah, that's one face; it's only five inches by five inches.

Clarissa: We've got it done—it's 25. 25 square inches?

Julio: For each face, but add them—no, multiply them 6 times.

Claire noticed that many pairs took notes as they worked to find key measurements. While they seemed to understand the net approach to finding surface area, some pairs needed to clarify which parts of their solids were the key sides to measure. Michael and Carlos had a small pyramid. As they worked, they had to identify key lengths before applying the formula $A = \frac{1}{2}bh$, the area of a triangle (Figure 2.10).

Carlos: So, what was the base again? Did we write it down?

Michael: Yeah, but we have to figure out how to get the area of these four sides. The triangle ones. And write those down to add up later (consults the formula sheet). Which one is the base of the triangle—the side edge or the base edge?

Carlos: (looking from formula sheet to the pyramid) Is it the height of the whole thing, the pyramid—or the height of each triangle, the triangle face?

Figure 2.10 Michael and Carlos Work on Calculating Surface Area of Their Pyramid

Leon and Cloe were working on a sphere. They had quickly realized that they needed to consult the formula sheet for a way to calculate the surface area of this solid. After some discussion, and use of the Math Terms & Symbols word wall, they discovered that the measurement they had taken around the middle of the sphere was the circumference, not the diameter. They knew they'd need to find the radius in order to find the surface area of their

sphere using the formula $SA = 4\pi r^2$. Frustrated because they couldn't cut their sphere in half (and annoyed that their friends Sofia and Alondro were having an easier time with their half sphere), Leon proposed a method for measuring radius.

Leon: Okay, you hold the sphere. Now if we made a tube around it—

Cloe: Tube? Where?

Leon: Like a cylinder, right around it. Tight. Then we could put a ruler here, the end of the cylinder, across it—that would be diameter. And divide by 2, that would be radius.

But Cloe, pictured in Figure 2.11, had another idea about how to calculate the radius.

Cloe: Well. But that way is harder. We don't need to make a cylinder. We just need radius. A way to get it. There was something—a way we did. . . .

Claire (who had been listening in, clipboard in hand) Good ideas, you two. You're looking for way to find the radius directly. But what did we discover when we did that lab with circumference of circles and pi? Is there a way to figure it out mathematically?

Cloe: Yes there was! See Leon—pi times diameter, or something (checks formula sheet). We can just do it on paper, like an equation, it's easier. The cylinder thing—it might not be exact, you know?

Claire: You can calculate the length of the radius based on what you know about other lengths. Or you can use a direct measurement process that you were exploring. Two possibilities.

Cloe: Yeah, but we don't have to measure with our ruler for radius. We can figure it out.

As she circulated around the room with her clipboard, Claire made sure to note evidence of her ELLs' use of comparing, contrasting, and explanatory language. She mostly listened, but occasionally interjected a comment or a question to have them reflect on the processes that they would be reporting to the whole class for surface area calculation.

The pairs finished their calculations at different rates. Claire had those who finished first go to the tissue paper station and begin cutting and wrapping their solids. One pair finished their calculations and wrapping quite quickly, and Claire had made sure that a short video about Pancho Quilici's 3-D art was cued up on her computer for early finishers to watch (Tablante, 2011).

Figure 2.11 Leon and Cloe Try Different Methods for Direct Measurement of a Sphere


After 20 minutes of pair work, Claire was confident that all the pairs had discovered a process for calculating surface area for their solids. Only a few pairs had wrapped their solids, those with rectangular prisms or cylinders, but Claire knew they could continue wrapping over the next few days. With only 20 minutes left in math block, Claire chose to invite each pair of students with an early advanced or advanced ELL to report to the class their processes for calculating surface area. To encourage use of complete sentences and contrasting language, Claire reminded each student who was reporting to "tell the class what was the easiest part of your process, and what was hardest, as part of your report." Claire listened and made more notes on individual student rubrics, as she stood with her clipboard.

Consider this . . .

Notice that Claire was assessing the students' English language proficiency; if she had been assessing their achievement in learning math content, she would be asking for the answers to their calculations. What is the benefit of integrating language assessment with content instruction?

To wind up her lesson, Claire invited a student to read the daily objectives again. She proudly announced "I think we've accomplished our goals today. Stand up if you think you can explain a possibility for calculating the surface area of a rectangular prism . . . of a cone . . . of a sphere." Most students jumped up when each solid was named. "Great. Your homework tonight is simple: Read the information on page 582 in your textbook about two methods for finding the surface area of a prism. Then write a word problem of your own for the class that requires us to find the surface area of a prism." Students had a "30-second cleanup" while Claire replayed the Rafael Barrios YouTube clip, and then they lined up for their buses.

As she reflected on that day's lesson, Claire felt that students' time working in pairs and discovering some of the inherent difficulties in direct measurement had been well spent. She was especially proud of those students who had worked on solids with curved faces, and the ways in which each pair had reported their processes to the class.

Consider this . . .

Claire incorporated various means of assessment into her lesson. Consider how you can collect and record formative data in order to share what you are learning about students with colleagues.

Following Up

The next day, Claire showed her students the fifth grade blog and then invited them all individually to blog about what they had discovered about calculating surface area the day before. She challenged students to use their notes and journals as a springboard to more real-world applications of geometry, noting that they would be doing some geometry blogging as part of language arts time in the next several weeks.

In their blog posts, Claire wanted her students to make use of the academic language surrounding geometric forms that they had learned: "I want you to use all your geometry language as you write," Claire said. "Use our word wall; use your own notes and journals that you have been keeping, and make sure the geometry language comes out! For example, as I watched you calculate surface area of spheres yesterday, it made me think about the work that cartographers have to do. Cartographers make maps, and think for a minute about the maps that are round—that are put onto globes. Those are spheres, right? And the flat drawing of the continents somehow has to be perfectly cut and shaped, with no overlapping, to wrap the surface of the globe. That would be a trick!"

"Yes," said Cloe thoughtfully. "You'd have to cut, like slices of the paper on each side—it would be a spiky flat paper to go around and cover a sphere perfectly." Claire smiled. Cloe was thinking hard about surface area of curved shapes.

As students reviewed the text's methods for calculating surface area, it was evident to Claire that their hands-on experience the day before was helping deepen their understanding of the text's problems, explanations, and geometry vocabulary. "This method with a net is like what we did with each face of our pyramid," mused one student as he reviewed page 582 of the text. And Sofia, using her math journal as a springboard, had posted in her English math blog:

> After hearing what people said about measuring their shapes yesterday I decided that's why mathematicians developed formulas for surface area, for when they couldn't measure something directly, like the inside of a sphere or the height of a huge building.

Claire decided that she would alert Eduardo to the value of having students do some hands-on discovery with volume of their solids next week, before they tackled the volume word problems in the text.

REFLECTION ON THE UNIT: LOOKING BACK AND MOVING FORWARD

Eduardo and Claire were pleased with the students' scores on their unit test and their homework, as well as their day-to-day demonstrations of geometric knowledge. The students in sixth grade had, indeed, used many of their covered shapes as part of piñatas for the whole school's Día de los Niños celebration. They knew exactly which shapes held the largest volume of candy, and some of the younger kids had actually asked about how the sixth graders had learned to calculate volume and how they knew how much wrapping paper they had used on different shapes in the piñatas. When the sixth graders had blogged about their piñata construction in English and in Spanish, several parents had responded positively to the posts and the pictures from Kids' Day that the students had attached on the school's bilingual website.

Claire and Eduardo felt they'd hit some important aspects of sixth grade Common Core State Standards for mathematics. Moreover, they had some good data on the developing English language proficiencies of their 11 ELLs, thanks to their use of the oral speaking checklists, their discussions of how the students were performing on math blogs, and Claire's own heightened awareness of all her students' use of comparing/contrasting language as they focused on mathematical processes. Claire wanted to continue to refine her standards-based recording forms with Eduardo; she had a new goal for next year to document language process in both languages at least once in every unit with a standards-based form. She thought it would be especially helpful as her whole team moved to Common Core State Standards, and she planned on sharing what she learned through the initial implementation at the next data team meeting.

Claire specifically acknowledged the way in which this unit helped in planning to assess academic language use around math content. She knew she had a long way to go, but she had great colleagues in her school who were also invested in documenting the language progress of middle school students. As she looked around her room, Claire saw a quote from Pancho Quilici (cited at the beginning of this chapter), highlighted in the *Américas* magazine article about geometric forms and possibilities. Yes, Claire thought, geometry—and all of math—was full of possibilities—for her as a developing teacher and for her students.

APPENDIX A
CLAIRE'S LESSON PLAN TEMPLATE

Grade Level: 6

Topic: Calculating Surface Area of Geometric Solids

Content Standard(s)

Sixth grade Common Core State Standards for geometry:

Geometry 6.G	H. Solve real-world and mathematical problems involving surface area

Oregon English Language Development Standards:

Language Function	*Early Advanced ELLs*	*Advanced ELLs*	*Grammatical Forms*
Comparing	Varied sentence structures with specific comparative adjectives and phrases.	Complex sentence structure with specific comparative language	Adjectives and Conjunctions
Contrasting	Subject/verb/adjective, *both* subject/verb, *but*	Approximately used idiomatic phrases and contrasting words (e.g., *whereas*, and *in contrast*)	Comparative Adjectives
Explaining	Get across important points using declarative, compound and complex sentences, regular and irregular verb forms.	Get across which point he/she feels is most important using regular and irregular verb forms, adverbs of manner and compound-complex sentences.	Verb Forms—Indicative verb *(makes a statement of fact)*. Declarative Sentences, Complex Sentences, Adverbs of Manner.

Source: Language Functions and Forms, n.d., pp. 7, 9.

Instructional Objectives

CONTENT: Students will calculate the surface area of a selected geometric solid in a small group.

ACADEMIC LANGUAGE: With a focus on use of the explanatory and comparative language, students will explain the process for calculating surface area of a selected solid to their peers.

Assessments Within This Lesson

- Teacher observation; teacher checks for understanding.
- On their pair worksheets, teacher checks students' calculations of the surface area of their selected solids.
- Teacher uses speaking rubric to assess ELLs' use of language during oral reports and during pair work.

Adaptations for Learners With Special Needs

- Visual support: updated word wall, math news board with related articles, featured texts in math library, geometry video clips, manipulatives (various solids), objectives posted, use of "thumbs up" for understanding and comprehension checks; teacher-created pair worksheets to provide direction for those who need it
- Differentiated language rubric for early advanced and advanced ELLs
- Deliberate pairing of students within zones of proximal development for today's pair work
- Teacher, parent volunteer circulating during work time for scaffolding understanding

Materials Needed

- Geometric solids in various sizes, measuring tools, tissue paper, glue, tape
- *Official Formula Sheet and Conversion Table,* Grades 6-8, available at http://www.ode.state.or.us/wma/teachlearn/testing/dev/testspecs/formulapage-gr6-8_2010-2011_july-7.pdf
- Teacher-developed pair worksheet: "How do we calculate the surface area of different geometric solids?" (see Appendix B)
- Students' math journals
- Smart board for use during student reports, teacher clarification, and playing YouTube video about sculptures
- Adopted math textbook for use as reference

Getting Started (Opener)

Teacher jumpstart: What is geometry about? Student input. Teacher shows YouTube video about huge geometric shapes created in work of Rafael Barrios.

Summary of the Lesson

What Will Teacher Do?	*What Will Students Do?*
Provide math journal prompt: Where do we see polyhedrons in our community?	Respond in writing in individual math journals
Outline daily objectives and post	One student presents objectives
Introduce shapes as each is pulled from bag; quickly review names of solids	Respond to teacher requests for shape identification
Model finding surface area of cube	Listen to and watch modeling
Circulate to observe student pair work; note evidence of targeted language use as pairs work together; provide scaffolding if needed as students work to calculate surface area	Work in pairs to calculate surface area of their selected solid. Plan to explain to class their process for calculation, noting hard and easy parts of the process. Use pair worksheet as needed. Begin work on covering solid with least amount of tissue paper possible, based on calculations, as time permits.
Select students to report process to class. (If time is limited, make sure all ELLs have a chance to report.)	Report their calculation processes to the class, using their solids and any notes they have prepared to demonstrate. Listen to others' reports. Respond to check for understanding with thumbs up or down. Ask questions if needed about assigned homework.

Closure

Direct students' attention back to daily objectives. Check for understanding of surface area calculation with thumbs up/down for each solid explored by pairs of students.

APPENDIX B
SHEET FOR PAIR WORK

How do we calculate the surface area of different geometric solids?

Our group members:_____

1. What kind of geometric solid do you have? Draw and name it below.

2. Does your geometric solid have straight lines and sides? If YES, carefully measure *each* side. Write your measurements here.

   Now use your measurements to calculate the surface area for your geometric solid. Write your process below.

3. Does your geometric solid have curves? If YES, which formula will you use to calculate the surface area of your geometric solid?

   Write the formula you will use here.

Now find the measurements you need to use the formula to calculate the surface area of your curved geometric solid. Write your process and results here.

4. Now that you know the surface area of your geometric solid, think: What is the *least* amount of wrapping paper you will need to cover your solid?

When you are ready to ask for the least amount of wrapping paper you'll need, come to the wrapping table to get your supplies! (Bring this paper with you to show your calculations.)

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Three Articles About Geometry Applications on the Math News Board

Snow, K. (2006). A mind in MOTION. *Américas, 58*(5), 16–23. A profile of the geometry-based artwork of Venezuelan artist Pancho Quilici.

Weiss, P. (2004). Squashed spheres set a record for filling space. *Science News, 165*(25), 397. A short news brief about shapes used in packing materials.

Wise, B. (2009). Sandy solids! *Scholastic Math, 29*(12), 10.

CLAIRE'S TOP TEN TRADE BOOKS THAT FEATURE GEOMETRY

Picture Books

Grant, J. (2006). *Cat and fish go to see.* Singapore: Simply Read Books. Neil Curtis' illustrations echo classical engraving with dramatic black and white movement.

Rowling, J. K. (1999). *Harry Potter and the prisoner of Azkaban*. New York, NY: Arthur A. Levine Books. Mary Grandpré's play with geometric shapes and line have delighted American readers of the Harry Potter series—Claire referred to her illustrations of the expanded Aunt Marge in this book.

Vamos, S. R. (2011). *The cazuela that the farm maiden stirred* (bilingual). Watertown, MA: Charlesbridge. Rafael López's vibrant shapes feature collage-type illustrations with radiant geometric shapes and colors.

Novels, Biographies, and Other Books Featured in Claire's Math Library During This Math Unit

Ball, J. (2005). *Go figure!* London, UK: DK.

D'Agnese, J. (2010). *Blockhead: The life of Fibonacci.* New York, NY: Henry Holt.

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