

# Why Is It Important for My Students to Learn Conceptually?

**A**round the world, mathematics is highly valued and great importance is placed on learning mathematics. Private tutors in non-Asian countries serve a remedial purpose, whereas in Asia, everyone has a tutor for providing an increased knowledge base and skill development practice. Many students in Asia enroll in programs like “Kumon,” which focus on practicing skills (which has its place) and “doing” math rather than “doing and understanding” math. When you ask students who are well rehearsed in skills to problem solve and apply their understanding to different contexts, they struggle. The relationship between the facts, skills, and conceptual understandings is one that needs to be developed if we want our students to be able to apply their skills and knowledge to different contexts and to utilize higher order thinking.

## Why Do We Need to Develop Curriculum and Instruction to Include the Conceptual Level?

According to Daniel Pink (2005), author of *A Whole New Mind*, we now live in the Conceptual Age. It is unlike the Agricultural Age, Information Age, or the Industrial Age because we no longer rely on the specialist content knowledge of any particular person. The Conceptual Age requires individuals to be able to critically think, problem solve, and adapt to new environments by utilizing transferability of ideas. “And now we’re progressing yet again—to a society of creators and empathizers, of pattern recognizers and meaning makers” (Pink, 2005, p. 50).

Gao and Bao (2012) conducted a study of 256 college-level calculus students. Their findings show that students who were enrolled in concept-based learning environments

scored higher than students enrolled in traditional learning environments. Students in the concept-based learning courses also liked the approaches more. A better grasp of concepts results in increased understanding and transferability.

With the exponential growth of information and the digital revolution, success in this modern age requires efficient processing of new information and a higher level of abstraction. Frey and Osborne (2013) report that in the next two decades, 47% of jobs in the United States will no longer exist due to automation and computerization. The conclusion is that we do not know what new jobs may be created in the next two decades. Did cloud service specialists, android developers, or even social marketing companies exist 10 years ago?

How will we prepare our students for the future? How will our students be able to stand out? What do employers want from their employees? It is no longer about having a wider knowledge base in any one area.

Hart Research Associates (2013) report the top skills that employers seek are the following:

- Critical thinking and problem solving,
- Collaboration (the ability to work in a team),
- Communication (oral and written), and
- The ability to adapt to a changing environment.

How do we develop curriculum and instruction to prepare our students for the future?

**W**e owe our students more than asking them to memorize hundreds of procedures. Allowing them the joy of discovering and using mathematics for themselves, at whichever level they are able, is surely a more engaging, interesting and mind-expanding way of learning. Those “A-ha” moments that you see on their faces; that’s why we are teachers.

David Sanda, Head of Mathematics  
Chinese International School, Hong Kong

## The Structure of Knowledge and the Structure of Process

Knowledge has a structure like other systems in the natural and constructed world. Structures allow us to classify and organize information. In a report titled *Foundations for Success*, the U.S. National Mathematics Advisory Panel (2008) discussed three facets of mathematical learning: the factual, the procedural, and the conceptual.

These facets are illustrated in the Structure of Knowledge and the Structure of Process, developed by Lynn Erickson (2008) and Lois Lanning (2013).

The **Structure of Knowledge** is a graphical representation of the relationship between the topics and facts, the concepts that are drawn from the content under study, and the generalization and principles that express conceptual relationships (transferable understandings). The top level in the structure is Theory.

**Theory** describes a system of conceptual ideas that explain a practice or phenomenon. Examples include the Big Bang theory and Darwin’s theory of evolution.

The **Structure of Process** is the complement to the Structure of Knowledge. It is a graphical representation of the relationship between the processes, strategies, skills and concepts, generalizations, and principles in process-driven disciplines like English language arts, the visual and performing arts, and world languages.

For all disciplines, there is interplay between the Structure of Knowledge and the Structure of Process, with particular disciplines tipping the balance beam toward one side or another, depending on the purpose of the instructional unit. The Structure of Knowledge and the Structure of Process are complementary models. Content-based disciplines such as science and history are more knowledge based, so the major topics are supported by facts. Process-driven disciplines such as visual and performing arts, music, and world languages rely on the skills and strategies of that discipline. For example, in language and literature, processes could include the writing process, reading process, or oral communication, which help to understand the author’s craft, reader’s craft, or the listener’s craft. These process-driven understandings help us access and analyze text concepts or ideas.



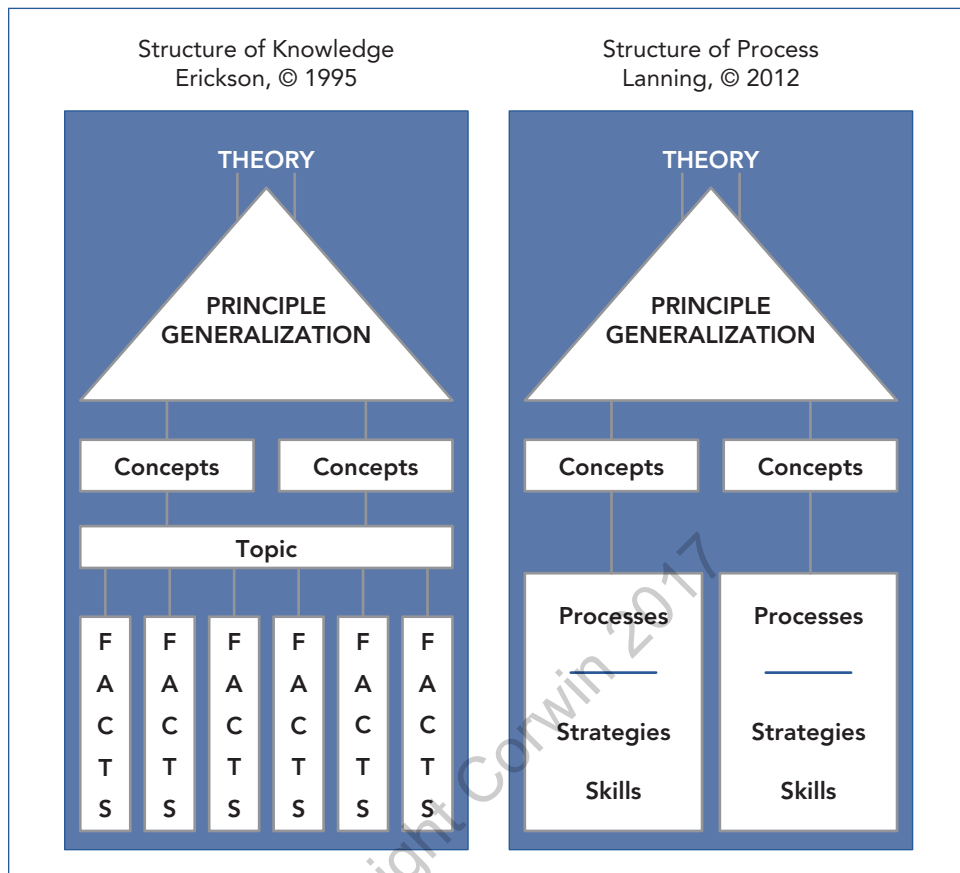
Both structures have concepts, principles, and generalizations, which are positioned above the facts, topics, or skills and strategies. Figure 1.1 illustrates both structures. Figure 1.1 can also be found on the companion website, to print out and use as a reference.

## The Structure of Knowledge and the Structure of Process for Functions

Mathematics *can* be taught from a purely content-driven perspective. For example, functions can be taught just by looking at the facts and content; however, this does not support learners to have complete conceptual understanding. There are also processes in mathematics that need to be practiced and developed that could also reinforce the conceptual understandings. Ideally it is a marriage of the two, which promotes deeper conceptual understanding. Figure 1.2 illustrates the Structure of Knowledge for the topic of functions.

**Topics** organize a set of facts related to specific people, places, situations, or things. Unlike history, for example, mathematics is an inherently conceptual language, so “Topics” in the Structure of Knowledge are actually *broader concepts*, which break down into micro-concepts at the next level.

FIGURE 1.1: SIDE BY SIDE: THE STRUCTURE OF KNOWLEDGE AND THE STRUCTURE OF PROCESS



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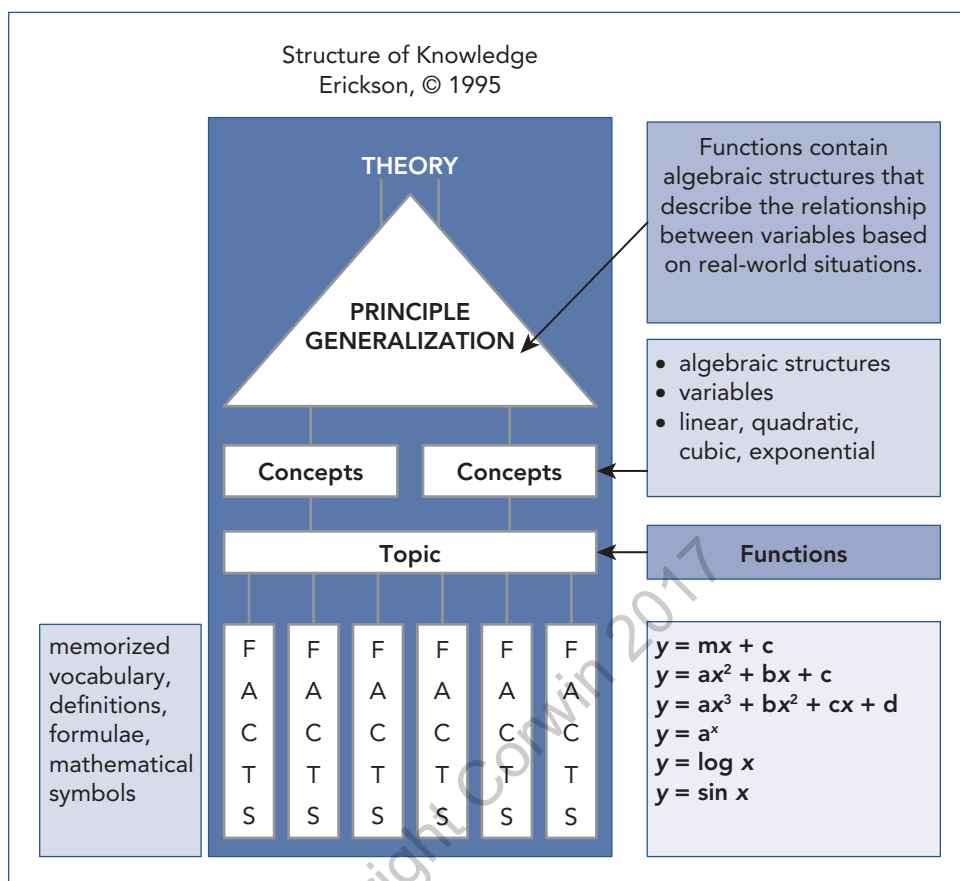
As explained by Lynn Erickson (2007), “The reason mathematics is structured differently from history is that mathematics is an inherently conceptual language of concepts, subconcepts, and their relationships. Number, pattern, measurement, statistics, and so on are the broadest conceptual organizers” (p. 30).

More about concepts in mathematics will be discussed in Chapter 2.

**Facts** are specific examples of people, places, situations, or things. Facts do not transfer and are locked in time, place, or a situation. In the functions example seen in Figure 1.2, the facts are  $y = mx + c$ ,  $y = ax^2 + bx + c$ , and so on. The factual content in mathematics refers to the memorization of definitions, vocabulary, or formulae. When my student knows the *fact* that  $y = mx + c$ , this does not mean she understands the *concepts* of linear relationship,  $y$ -intercept, and gradient.

According to Daniel Willingham (2010), automatic factual retrieval is crucial when solving complex mathematical problems because they have simpler problems

FIGURE 1.2: THE STRUCTURE OF KNOWLEDGE FOR FUNCTIONS



Adapted from original Structure of Knowledge figure from *Transitioning to Concept-Based Curriculum and Instruction*, Corwin Press Publishers, Thousand Oaks, CA.

embedded in them. Facts are the critical content we wish our students to know, but they do not themselves provide evidence of deep conceptual understanding.

Formulae, in the form of symbolic mathematical facts, support the understanding of functions. This leads to a more focused understanding of the *concepts* of linear functions, quadratic functions, cubic functions, exponential functions, variables, and algebraic structures in Figure 1.2. The generalization “Functions contain algebraic structures that describe the relationship between two variables based on real-world situations” is our ultimate goal for conceptual understanding related to the broad concept of functions. Please take a look at the companion website for more examples of the Structure of Knowledge and the Structure of Process on the topic of linear functions. See Figures M1.1 and M1.2.



**Concepts** are mental constructs, which are timeless, universal, and transferable across time or situations. Concepts may be broad and abstract or more conceptually specific to a discipline. “Functions” is a broader concept, and the micro-concepts at the next level

are algebraic structures, variables, linear, quadratic, cubic, and exponential. Above the concepts in Figure 1.2 are the principles and generalizations.

**Generalizations are statements that connect two or more concepts.**

**Principles** and **generalizations** are transferable understandings that allow students to make connections between two or more concepts. In mathematics, the principles are the theorems, the cornerstone truths. Though generalizations and principles are both statements of conceptual relationship, the principles do not contain a qualifier such as *often*, *can*, or *may* because they are immutable “truths” as we know them. Because generalizations do not rise to the level of a law or theorem, they *may* require a qualifier if they do not hold true in all cases. Principles and generalizations are often exemplified in a real-life context for mathematics; however, they are not exclusively portrayed in this way. In Figure 1.2, another generalization could have been the following: “Algebraic tools allow highly complex problems to be solved and displayed in a way that provides a powerful image of change over time” (Fuson, Kalchman, & Bransford, 2005, p. 351).

Although the Structure of Knowledge provides the deep understanding of the content of mathematics, the processes, strategies, and skills also provide important conceptual understanding.

The Structure of Process represents the procedural facet of learning mathematics. Processes, skills, and strategies are included in the lowest levels in the Structure of Process. “Skills are smaller operations or actions that are embedded in strategies, and when appropriately applied ‘allow’ the strategies to work. Skills underpin a more complex strategy” (Lanning, 2013, p. 19).

**Strategies** are systematic plans that learners consciously adapt and monitor to improve learning performance. As explained by Erickson and Lanning (2014), “Strategies are complex because many skills are situated within a strategy. In order to effectively employ a strategy, one must have control over a variety of the skills that support the strategy.” (p. 46). An example of a strategy in math would be making predictions or drawing conclusions.

**Processes** are actions that produce results. A process is continuous and moves through stages during which inputs (materials, information, people’s advice, time, etc.) may transform or change the way a process flows. A process defines what is to be done—for example, the writing process, the reading process, the digestive process, the respiratory process, and so on.

Figure 1.3 illustrates an example of the mathematical process of creating representations and the generalizations associated with this mathematical process. Throughout this functions unit, students will learn different strategies and skills that support the process of creating representations. This could include using a table of values or an algebraic or geometric form of a function.

Concepts that can be drawn from this process include substitution, revision, interpretation, and models. Two or more of the concepts are used to write unit

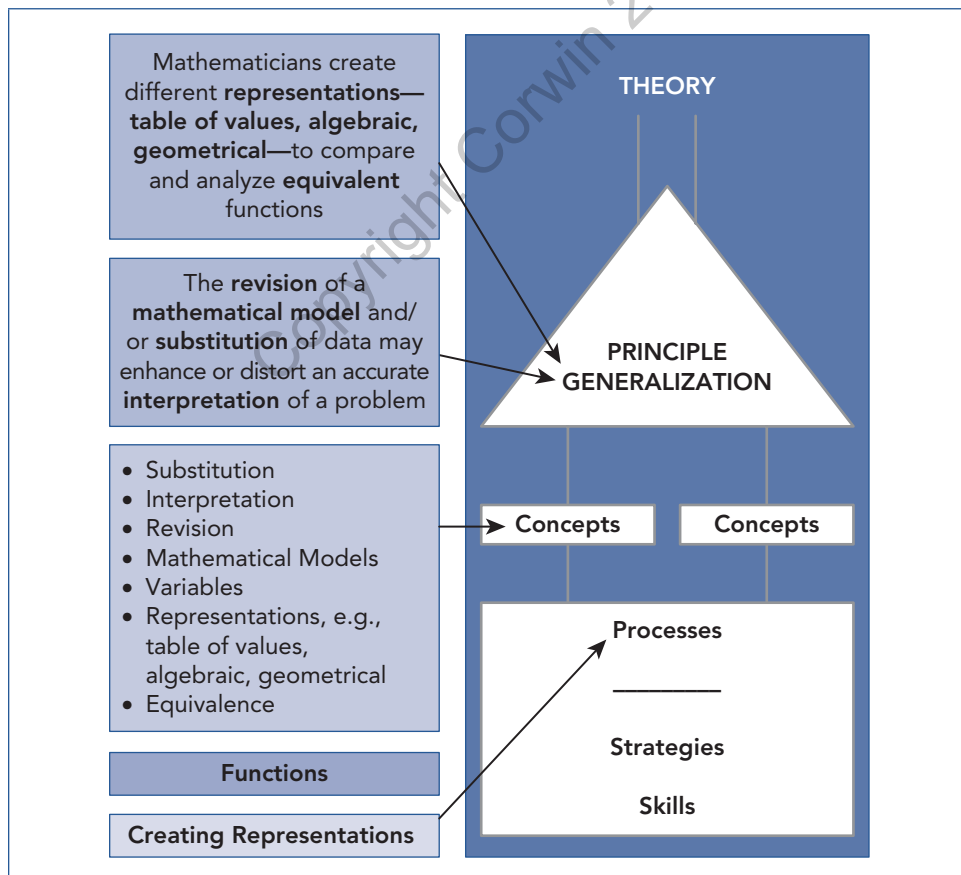
generalizations, which are also known as process generalizations. The process generalizations in Figure 1.3 are as follows:

**M**athematicians create different representations—table of values, algebraic, geometrical—to compare and analyze equivalent functions.

**T**he revision of a mathematical model or substitution of data may enhance or distort an accurate interpretation of a problem.

When students are guided to these generalizations, they demonstrate their understanding of the creating representations process.

FIGURE 1.3: STRUCTURE OF PROCESS EXAMPLE FOR FUNCTIONS



Adapted from original Structure of Process figure from *Transitioning to Concept-Based Curriculum and Instruction*, Corwin Press Publishers, Thousand Oaks, CA.

Other strategies and skills, such as graphing and analytical skills, support the process of creating representations. This process supports the concepts of mathematical models, substitution, interpretation, revision, variables, equivalence, and so on.

In Figure 1.4, we look at the dual part that the Structure of Knowledge and the Structure of Process each play in ensuring a deep understanding of content and process in mathematics. For the concept of functions, we include the content that needs to be learned as well as the skills and strategies that are employed fluently to aid the process of creating representations. The ability to employ strategies and skills fluently is referred to as *procedural fluency*. Visit the companion website to see additional summaries of the components of the Structures of Knowledge and Process. See Figures M1.3 and M1.4.



To help understand the generalization “Functions contain algebraic structures that describe the relationship between two variables based on real-world situations,” we work to ensure the conceptual relationships are revealed. The concepts of algebraic structures, variables, linear, quadratic, and cubic help us connect the facts to give mathematical content more meaning and promote deeper understanding. The mathematical process involved is creating representations, and it supports the understanding of the concepts substitution, interpretation, revision, variables, mathematical models, and equivalence. Mathematical processes will be discussed in detail in the next chapter.

The language of mathematics is different to languages like English and Chinese. There are things that are strictly allowed and there are things that are strictly not. It is the formal nature of the language that often causes confusion and errors in learners. However, over-emphasis on the formality, and some teachers are only concerned with practicing formal exercises, prevents understanding of the beauty, creativity, and utility of mathematics.

Chris Binge, Principal Island School, Hong Kong

## Applying the Structure of Knowledge and the Structure of Process

### Inductive vs. Deductive Teaching

In my first years of teaching, it was common practice in the mathematics classroom to adopt the PPP model (presentation, practice, and production) of **deductive, teacher-led instruction**. The PPP approach typically looks like this:

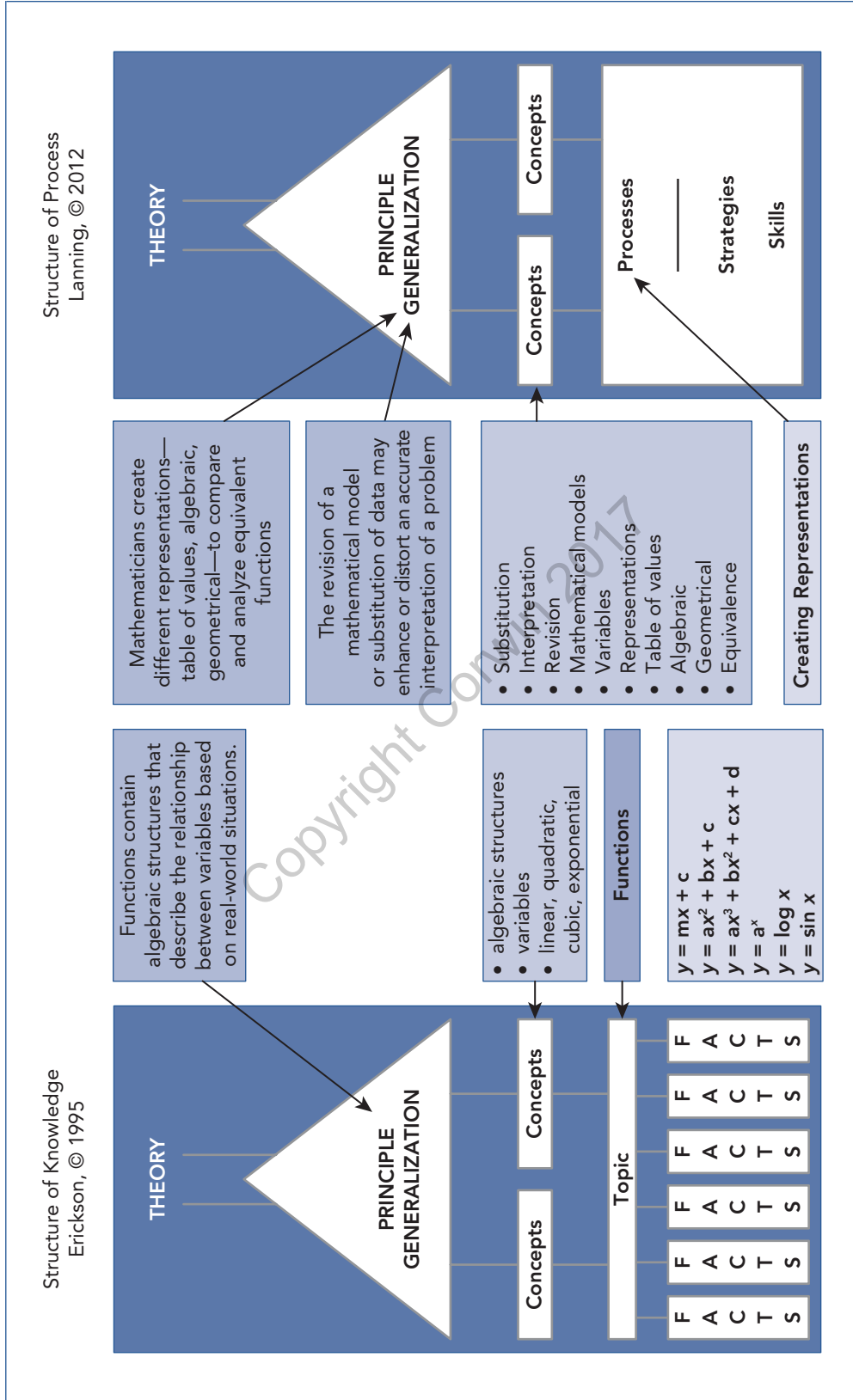
Step 1: Teacher introduces the formula, such as the Pythagorean theorem, and demonstrates three working examples.

Step 2: Ask students to practice using the formula.

Step 3: Ask students to produce their own examples.



FIGURE 1.4: THE STRUCTURE OF KNOWLEDGE AND THE STRUCTURE OF PROCESS FOR THE TOPIC FUNCTIONS, SIDE BY SIDE



Adapted from original Structure of Knowledge and Structure of Process figures from *Transitioning to Concept-Based Curriculum and Instruction*, Corwin Press Publishers, Thousand Oaks, CA.

The **two-dimensional model of instruction**, which focuses on the facts and content of the subject and the rote memorization of procedures and topics, is intellectually shallow. A two-dimensional curriculum and instruction model focuses on the bottom levels of the Structure of Knowledge and the Structure of Process. This encourages students to work at a low-order level of thinking (such as memorization of facts or perfunctory performance of lower level skills) in a content/skill-based, coverage-centered curriculum. A two-dimensional model often presents the generalization or new concepts at the beginning of the learning cycle and follows a direct teaching methodology.

This is typical of a deductive approach in teaching. I have witnessed many, many lessons utilizing this approach, and to me, this is like telling our students what the present is before they open it! The concept-based model is generally an inductive teaching model that draws the understandings from the students as a result of structured or guided inquiry.

An **inductive approach**, like mathematical induction, allows learners to start with specific examples and form generalizations for themselves. In his research on how the brain learns mathematics, David Sousa (2015) states that the human brain is a powerful pattern seeker, and we have an innate number sense or what scientists call “numerosity.” The inductive approach utilizes this innate quality for number sense and pattern finding. The teacher acts as a facilitator, helping students to discover relationships and seek patterns for themselves.

Inductive approaches lead to generalization formation.

The **three-dimensional model of instruction** suggests a more sophisticated design with a third level: the conceptual level. In a three-dimensional curriculum and instruction model, the lower levels of the Structure of Knowledge and the Structure of Process are important components, but the third dimension of concepts, principles, and generalizations ensures that conceptual thinking and understanding are prominent.

A three-dimensional, inductive approach encourages students to construct generalizations at the end of the learning cycle through the use of inquiry. As stated by Erickson and Lanning (2014), “Deep understanding and the transfer of knowledge and skills require that teachers understand the relationship between the factual/skill level and the conceptual level, and use this relationship effectively in instruction” (p. 23).

Figure 1.5 illustrates the difference between inductive and deductive approaches.

**FIGURE 1.5: INDUCTIVE VS. DEDUCTIVE APPROACHES**

<b>Deductive Approach</b>	Students are given the generalizations at the beginning of a lesson	Students then practice the generalizations through specific examples
<b>Inductive Approach</b>	Students are given specific examples at the beginning of the lesson	Students construct generalizations from

An inductive model is a student-centered approach, helping students to think logically and scientifically and allowing students to generalize by utilizing higher order thinking. Discovering inductive approaches changed my entire teaching practice and influences every student learning experience I plan for my students. The inductive approach provides a framework; it is a structure for all mathematical concepts to be conveyed to students in an analytical, coherent fashion. The key to inductive teaching is that students draw and form generalizations by working on specific examples initially.

Introducing the Pythagorean theorem utilizing an inductive approach would look like this:

1. Look at the following right-angled triangles and work out the squares of each of the sides. (Students work out specific numerical examples.)
2. What generalization can you make about the relationship between all three sides when they are squared? (Students now generalize by pattern seeking.)

Bransford, Brown, and Cocking (2000) offer a comprehensive survey of neurological and psychological research that provides strong support for constructivism and inductive methods. “All new learning involves transfer of information based on previous learning” (p. 53).

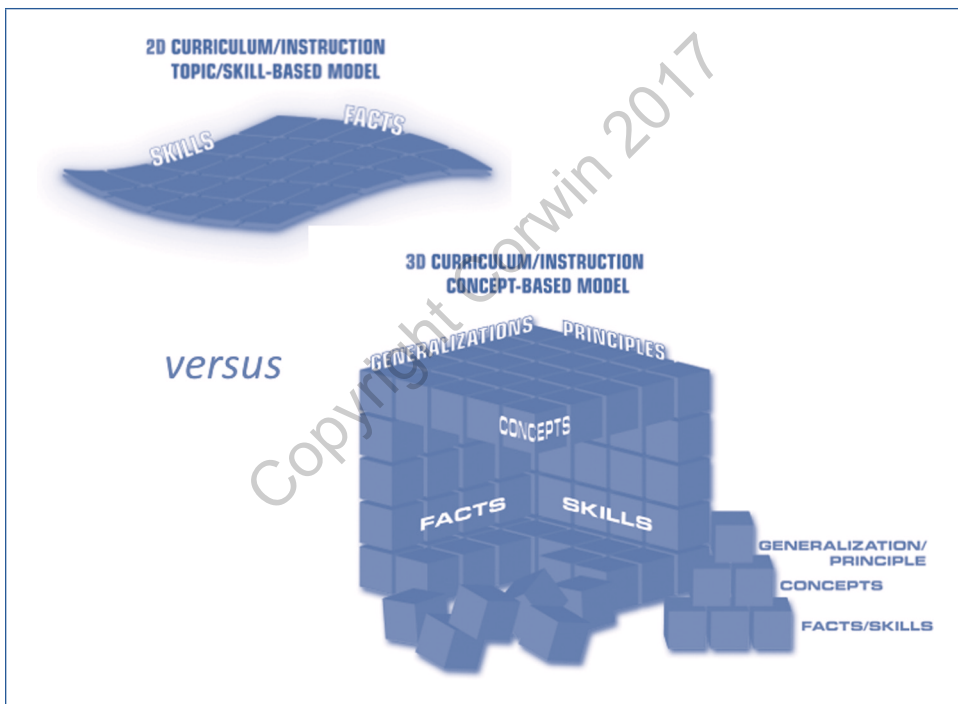
Inductive instruction presents new information in the context of situations, issues, and problems to which students can relate, so there is a much greater chance that the information can be linked to their existing cognitive structures. John D. Bransford et al. (2000) explain, “Motivation to learn affects the amount of time students are willing to devote to learning. Learners are more motivated when they can see the usefulness of what they are learning and when they can use it to do something that has an impact on others” (p. 61).

Inductive methods, such as problem-based learning, support techniques that use authentic situations and problems.

Generalizations and principles in the Structure of Knowledge and the Structure of Process are timeless, universal, transcend cultures, and are transferable ideas. They allow the learner to connect the facts and concepts for deeper meaning and understanding. The three-dimensional model of curriculum and instruction, according to Erickson and Lanning (2014), includes concepts, generalizations, and principles to ensure that curriculum and instruction focus on intellectual depth, the transfer of understanding, and the development of conceptual brain schemata. The three-dimensional model is contrasted with the traditional two-dimensional model of coverage and memorization.

Figure 1.6 illustrates the two-dimensional model, also known as the “inch deep, mile wide” approach to curriculum. In contrast, the three-dimensional model represents a more comprehensive, sophisticated design for curriculum and instruction.

**FIGURE 1.6: TWO-DIMENSIONAL VS. THREE-DIMENSIONAL CURRICULUM/INSTRUCTION MODELS**



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## Teaching for Inquiry

Inquiry is a vehicle and is about not telling students what the surprise is before opening the present. I have met many teachers in my travels, and often I hear the following about inquiry:

“I don’t have time for inquiry! I need to get through the content!”

“I have inquiry lessons once per week!”

“Inquiry just doesn’t work with my students; they need to be spoon fed!”

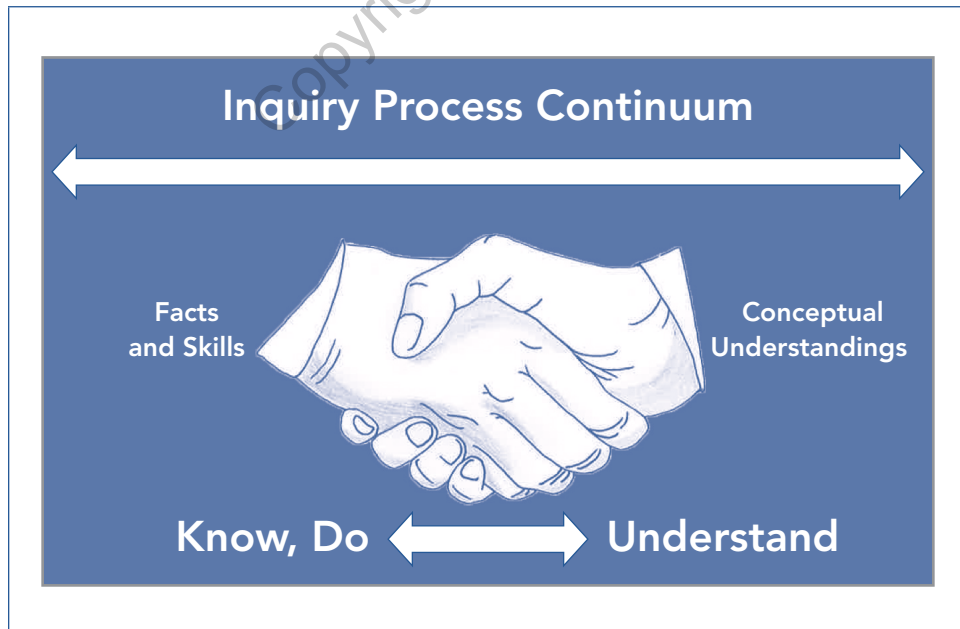
“Inquiry does not work for my students; they do not have the ability!”

Inquiry refers to posing questions, problems, or scenarios rather than providing established facts or knowledge. Inquiry means to seek truth, information, or knowledge, and individuals carry out the natural process of inquiry throughout their lives. Unfortunately, traditional curriculum discourages inquiry; students learn not to ask questions and to accept facts that are given. A study by Gelman, Gruber, and Ranganath (2014) found that learning is more effective when students are curious. Memory is also enhanced when students are in a state of curiosity. Inquiry encourages curiosity in students by posing questions to engage thought and interest.

Through inquiry and a variety of pedagogical approaches, such as cooperative and problem-based learning, students can develop skills for success while understanding the concepts involved (Barron & Darling-Hammond, 2008). Lynn Erickson encapsulates this idea as follows: “Information without intellect is meaningless.” Figure 1.7 illustrates the synergistic relationship between the facts, skills, and concepts all being achieved through a continuum of inquiry.

In order to develop intellect in our students we need to establish synergistic thinking through the inquiry continuum.

FIGURE 1.7: DEVELOPING INTELLECT THROUGH INQUIRY PROCESS CONTINUUM MODEL



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Erickson and Lanning (2014) state that “Synergistic thinking requires the interaction of factual knowledge and concepts. Synergistic thinking requires a deeper level of mental processing and leads to an increased understanding of the facts related to concepts, supports personal meaning making, and increases motivation for learning” (p. 36).

The vehicle of inquiry is used to foster synergistic thinking. The design of guiding questions in the form of factual, conceptual, and debatable questions also supports synergistic thinking and allows students to bridge the gap between the facts and skills and conceptual understandings.

For additional resources, visit the companion website where you will find an example of a traditional activity as well as guidance on how to facilitate synergistic thinking and a template to plan a synergistic student activity of your own. See Figures M1.6 & M1.7.



As an example, in order to understand the concepts of linear functions, parameters, and variables, one must know facts, such as  $y = mx + c$  or  $Ax + By + C = 0$ , and be able to plot points and create different representations. The inquiry process would ask students to investigate linear functions for different values for the parameters  $m$  and  $c$ . This supports the understanding of the concepts of linear, parameters, variables, and functions. Inquiry also stimulates student motivation and interest and leads to a deeper understanding of transferable concepts.

I have had the pleasure of working with Mike Ollerton, a pioneer in inquiry-based learning of mathematics from the United Kingdom. In his short piece on “Enquiry-Based Learning” (2013) he writes, “The underpinning pedagogy of enquiry-based learning (EBL) is for learners to gain and to use & apply knowledge in ways which places responsibility for the learning upon students. This is at the heart of supporting independent learning and requires the teacher become a facilitator of students’ knowledge construction; as a key aspect of sense making.”

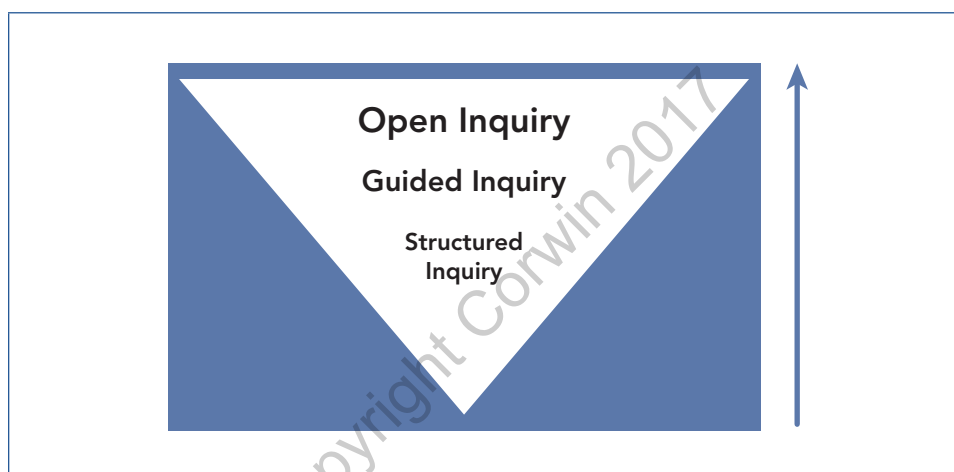
Different levels of inquiry are used as appropriate to the context and classroom situation. Figure 1.8 describes the levels of inquiry, adapted from the work of Andrew Blair (<http://www.inquirymaths.com>). Figure 1.9 shows the hierarchy of the levels of inquiry. The triangle represents the progression of inquiry levels, which can start off being quite narrow and structured, then move to a guided approach, and then ultimately to open inquiry, giving all students more opportunities to explore.

The three levels of inquiry—structured, guided, and open—originated in the learning approaches of science-based disciplines (Banchi & Bell, 2008). The important questions here are why and when do we use the different levels of inquiry?

FIGURE 1.8: LEVELS OF INQUIRY

Level	Description
Structured	<ul style="list-style-type: none"> <li>• Heavily scaffolded</li> <li>• Predictable line of inquiry</li> <li>• Predictable outcomes</li> </ul>
Guided	<ul style="list-style-type: none"> <li>• Different lines of inquiry</li> <li>• Predictable outcomes</li> </ul>
Open	<ul style="list-style-type: none"> <li>• Different lines of inquiry</li> <li>• Unpredictable outcomes</li> </ul>

FIGURE 1.9: LEVELS OF INQUIRY HIERARCHY



**Structured inquiry** is heavily scaffolded and suitable perhaps for learners and teachers who are new to inquiry. Structured inquiry fosters confidence in learners while promoting autonomy and independence. Teachers who are not accustomed to using inquiry find it difficult to “let go” of control, and structured inquiry provides a happy medium. The outcomes are predictable and predetermined by the design of the task.

**Guided inquiry** presents learners with opportunities for different lines of inquiry, with predictable outcomes. For example, ask students for different methods to prove a particular theorem (e.g., the Pythagorean theorem). Guided inquiry has fewer prompts and gives the learner more freedom to choose his or her own pathways to the desired outcome.

**Open inquiry** promotes different lines of inquiry with unpredictable outcomes. Truly authentic, open inquiry engages the learner’s interest and creativity. For

example, the International Baccalaureate Mathematics Standard and Higher Levels include an internal assessment called a “personal exploration.” Students are asked to choose an area of mathematics, conduct their own research, and draw their own conclusions. One of my past students, who was a ranked Hong Kong tennis champion, chose to write about tennis and binomial theorem. Another student with scoliosis looked at the curvature of her spine over the years using statistical analysis.

Open inquiry is not to be confused with pure “discovery” learning, when very little guidance is given to the learner. There is a misconception that inquiry is about giving students an open problem and letting them “run with it” with little guidance or input from the teacher. This is far from the intention of inquiry. Inquiry is student centered, inherently inductive, and peaks students’ motivation and interest. Inquiry is not an excuse for passivity. The teacher’s role is vital in facilitating and guiding the students during different stages of learning.

On the following pages there are three examples of student tasks on the same topic: proving the Pythagorean theorem. The topic is presented in three different ways to illustrate structured, guided, and open levels of inquiry.

Open inquiry is student centered, with extensive input from the teacher.

Figure 1.10 summarizes the main features and the difference between the three levels of inquiry for the Pythagorean theorem task. Figures 1.11, 1.12, and 1.13 are the student tasks.

FIGURE 1.10: LEVELS OF INQUIRY FOR PROVING THE PYTHAGOREAN THEOREM TASK

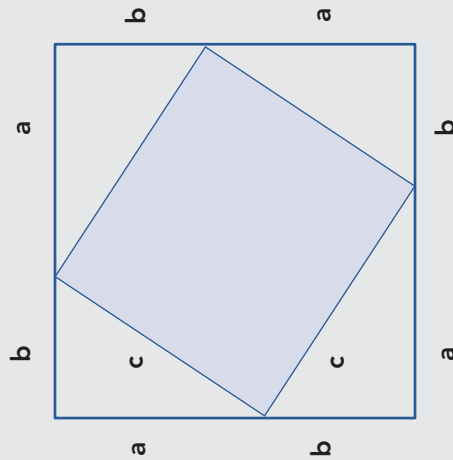
Task: Proving the Pythagorean Theorem	Features
Structured approach	Step-by-step scaffolded questions and prompts The table allows students to calculate the areas of different shapes within the large square and prompts students to find a relationship.
Guided approach	Fewer scaffolded prompts Given the large square with the tilted square inside, students must work out that finding the areas of the shapes inside.
Open approach	Students are asked to research their own proof with hundreds to choose from. They need to explain and show understanding of their proof.



FIGURE 1.11: A STRUCTURED INQUIRY EXAMPLE

### Proving Pythagorean Theorem

Find the area of the following shapes and complete this table.



A	B	C	B + C	Connecting A, B, and C
Area of the large square	Area of the tilted square	Area of the four triangles		

Explain in words the relationship you have discovered. Use a diagram to illustrate your explanation.



For a completed version of Figure 1.11, please visit the companion website.

## ●●● Sample Student Learning Experience

FIGURE 1.12: A GUIDED INQUIRY EXAMPLE

### Proving the Pythagorean Theorem

Investigate the relationship between  $a$ ,  $b$ , and  $c$  using the following diagram.

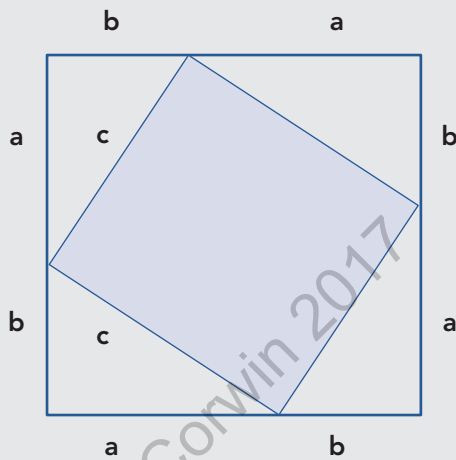


FIGURE 1.13: AN OPEN INQUIRY EXAMPLE

### Proving the Pythagorean Theorem

There are hundreds of proofs for the Pythagorean theorem. Research one proof and explain the proof with diagrams. Use any medium to explain your proof. This could include a poster, movie, applet, or Google presentation.

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To state a theorem and then to show examples of it is literally to teach backwards.

E. Kim Nebeuts From *Inspirational Quotes, Word, Sayings* (2015)

Through inductive inquiry, students are given opportunities to find generalizations and patterns they observe from specific examples. Studies have shown that a concept-based curriculum using an inductive approach results in a higher level of retention and conceptual understanding of the content.

Deductive approaches are the norm in traditional math classrooms—we rote-learn processes in a mechanical way without understanding the true reasoning and meaning behind the problem itself. Inquiry-based learning requires us to think and analyze for ourselves, then come up with a conclusion or generalization, which is the fun and beauty behind learning mathematics. We are encouraged to challenge ourselves and step away from our comfort zones in order to expand our knowledge of mathematics. Both learning methods are effective in the short term for an exam. But I have found inductive, inquiry-based approaches allow new information and working methods to be stored in my long-term memory as I actually understand what I am doing.

Chun Yu Yiu, Grade 12 student  
Island School, Hong Kong

According to Borovik and Gardiner (2007, pp. 3–4), the following are some of the top traits of mathematically able students:

- Ability to make and use generalizations—often quite quickly. One of the basic abilities, easily detectable even at the level of primary school: after solving a single example from a series, a child immediately knows how to solve all examples of the same kind.
- Ability to utilize analogies and make connections.
- Lack of fear of “being lost” and having to struggle to find one’s way through the problem.

Notice these abilities are described as traits that are not genetic predispositions but qualities that can be nurtured and developed in students. Opportunities to fail or “get stuck” give students the ability to lack fear of being lost or “stuck.” In her 2008 Harvard commencement address, J. K. Rowling, author of the *Harry Potter* books, said, “It is impossible to live without failing at something, unless you live so cautiously that you might as well not have lived at all—in which case, you fail by default.”

There are three principles outlined in the report *How Students Learn: Mathematics in the Classroom* (Bransford et al., 2005) that are consistent with the concept-based curriculum model:

Principle 1: Teachers must engage students' preconceptions. (p. 219)

This refers to recognition of students' prior knowledge and prior strategies and the need to build on them to create new strategies and new learning.

Principle 2: Understanding requires factual knowledge and conceptual frameworks. (p. 231)

This principle suggests the importance of the factual and conceptual and providing a framework for learners to connect the two in the form of generalizations. Learners need to have procedural fluency as well as know the conceptual relationships in order to develop mathematical proficiency.

Principle 3: A metacognitive approach enables student self-monitoring. (p. 236)

Learners need to be given time and space to explore mathematical concepts—in other words, to self-monitor. More opportunities to reflect on their experiences will help learners to construct their ideas into larger categories and take control of their own learning.

With this overwhelming evidence, you may now ask, how do we develop curriculum and instruction using a concept-based and inquiry-led model? In Chapter 2, we will look at the facts, skills, and strategies in mathematics and how to use them to build conceptual understanding through the Structure of Knowledge and the Structure of Process. Subsequent chapters provide practical activities to guide your journey in developing a three-dimensional concept-based model for curriculum and instruction.

**N**orthside ISD (San Antonio, TX) has been involved in concept-based curriculum for 10 years. It was important for this district that serves 103,000 students to have a K–12 curriculum in all major content areas that was developed using the tenets of concept-based curriculum. Our curriculum staff have been trained and certified by Lynn Erickson. Our teachers and administrators are clear about what our students are expected to know, understand, and do. Concept-based curriculum is without a doubt one of the main reasons Northside ISD continues to be a high performing district.

Linda Mora, Deputy Superintendent for Curriculum  
and Instruction Northside ISD,  
San Antonio, Texas

## Chapter Summary

This chapter laid the foundation for why we need to move from a two-dimensional to a three-dimensional curriculum and instruction model to include the conceptual level. Evidence supports the effectiveness of a concept-based curriculum, which is grounded in an inductive and inquiry-led approach. Concept-based models lead to increased mathematical proficiency and understanding. The chapter discussed what a concept-based curriculum looks like for math and the benefits to students' learning. An overview of the symbiotic relationship between the Structure of Knowledge and the Structure of Process in the realm of mathematics was also provided. Developing intellect requires synergistic thinking, which, according to Lynn Erickson (2007), is an interplay between the factual and conceptual levels of thinking. Synergistic thinking is at the heart of a concept-based curriculum and instruction.

An inductive model is a student-centered approach, helping students to think logically and scientifically, allowing students to generalize by utilizing higher order thinking. The inductive approach provides a framework; it is a structure for all mathematical concepts to be conveyed to students in an analytical, coherent fashion. The key to inductive teaching is that students draw and form generalizations by working on specific examples initially.

Levels of inquiry provide teachers and learners with the opportunity to gain confidence when exploring mathematical concepts. Structured and guided inquiry facilitates differentiation and promotes student and teacher confidence.

Extensive studies in mathematics education indicate a need for curriculum and instruction to include the conceptual level for enduring, deeper understandings. If we are to prepare our students for an unknown future, due to vast technological advances, we must ensure we foster higher order thinking skills.

The next chapter will explain, in detail, the Structure of Knowledge and the Structure of Process as applied to the facts, skills, strategies, and processes of mathematics.

## Discussion Questions

1. Does math education need to undergo a reform? Why or why not?
2. Why do educators need to include the conceptual understandings of a topic represented in a three-dimensional curriculum model?
3. How do the Structures of Knowledge and Process apply to the mathematics realm?
4. What are the features of inductive teaching and the benefits of an inductive approach when learning mathematics?
5. How does synergistic thinking develop intellect?
6. How would you use the different levels of inquiry in your classroom? Think of examples of when you might use each (structured, guided, open).

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