# MINE THE GAP FOR MATHEMATICAL UNDERSTANDING

# THE BOOK AT-A-GLANCE

**A quick-reference matrix provides a snapshot of the Big Ideas in the book, along with descriptions of the associated tasks.**

## BIG IDEAS & TASKS AT-A-GLANCE



| Big Idea No. | Big Idea | Task No. | Description |
|---|---|---|---|
| 1 | Adding within 1,000 | 1A | Students represent addition of three-digit addends on a number line. |
| 1 | Adding within 1,000 | 1B | Students add three-digit numbers represented with base ten blocks. |
| 1 | Adding within 1,000 | 1C | Students consider if three-digit addends can be decomposed by place value and added partially. |
| 1 | Adding within 1,000 | 1D | Students represent three-digit addition on a modified hundred chart. |
| 2 | Reasoning about addition within 1,000 | 2A | Students consider if adjusting addends for friendlier computation always works. |
| 2 | Reasoning about addition within 1,000 | 2B | Students compare sums by reasoning about the relationship between addends in two different expressions. |
| 2 | Reasoning about addition within 1,000 | 2C | Students create new expressions that will have a sum equal to a given expression. |
| 2 | Reasoning about addition within 1,000 | 2D | Students find addends for a given three-digit sum. |
| 3 | Subtraction within 1,000 | 3A | Students represent three-digit subtraction with four different representations. |
| 3 | Subtraction within 1,000 | 3B | Students represent subtraction with three-digit numbers on number lines. |
| 3 | Subtraction within 1,000 | 3C | Students are asked to break apart one or both numbers to make the subtraction of large numbers friendlier. |
| 3 | Subtraction within 1,000 | 3D | Students represent subtraction with three-digit numbers using a modified hundred chart. |

CHAPTER 2

# ADDITION AND SUBTRACTION WITHIN 1,000

*Chapter Overviews* highlight and explain the Big Ideas covered in each chapter.

**THIS CHAPTER HIGHLIGHTS HIGH-QUALITY TASKS FOR THE FOLLOWING:**

- Big Idea 1: Adding Within 1,000

  Multi-digit addition can be represented with different models, including place value models and number lines. Work with these models builds understanding and lays the foundation for flexible strategies for addition.

- Big Idea 2: Reasoning About Addition Within 1,000

  There is a relationship between addends and sums. Sums change as addends are changed. We can manipulate addends to make addition more friendly. Although addition strategies always work, the efficiency of the strategy relates to the numbers in the situation and the individual's own number sense.

- Big Idea 3: Subtraction Within 1,000

  Multi-digit subtraction can also be represented with place value models and number lines. Subtraction can be thought of as taking away, breaking apart, or comparing two values. We can count back (subtract) or count up (add) to find differences.

- Big Idea 4: Reasoning About Subtraction With

  Reasoning about subtraction situations can this, we need to understand the relationship b ences. As with addition, we can manipulate n

- Big Idea 5: Problem Solving With Addition an

  Problems can be thought of as any situation i addition and subtraction to solve problems. S of problems we encounter in mathematics or sense of the problem, knowledge of strategie

*Mining Hazard* icons signal examples of incomplete thinking that students may encounter.



BIG IDEA 1

## BIG IDEA 1
## Adding Within 1,000

Each Big Idea starts by describing one related high-quality task.

### TASK 1A

**Use the number lines to add 358 + 453. Use the number line to show how you added. Add 371 + 361. Use the number line to show how you added.**

### About the Task

We can represent addition with larger numbers on number lines. However, the size of these numbers limits the model to an open or empty number line. *These number lines do not have tick marks for each number.* In some cases, these number lines do not have defined endpoints either. In this task, students add different three-digit numbers on an open number line. The openness allows them to apply flexible strategies to the computation.

The highlighted task is explained in depth and potential student responses are predicted and described in detail.

### Anticipating Student Responses

Students are likely to decompose one or both numbers by their place value. For 358 + 453, these students may begin with 358 and make a jump of 400 (to 758), a jump of 50 (to 808), and then of 3 (to 811). *Some students may decompose an addend and then make repeated jumps of the place value. In other words, a jump of 400 would be represented by four jumps of 100.* Other students may jump by place value but begin with the ones place. They would first make a jump of 3, then 50, and then 400. Some students may make endpoints of 0 and 1,000 on their number line. Other students may assign one of the addends to the left endpoint and then jump/count on from there. Some of our students may find the sum of the numbers with an algorithm or similar procedure and then represent the addends and sum on the number line.

**MINING TIP**

It is important to connect ticked number lines with open number lines to support our students' transition to open number lines. We can adjust the intervals of tick marks to support the transition. For example, we can change the intervals from 1, 2, 3 to 10, 20, 30 or 100, 200, 300.

**MINING HAZARD**

Students who make a jump of 400 as four jumps of 100 are mathematically accurate. However, this strategy is less efficient than making one jump of a larger amount.

*Pause and Reflect* sections invite teachers to think about the task in relation to their practice and their own students.

**PAUSE AND REFLECT**

- How Does this Task Compare to Tasks I've Used?
- What Might My Students Do In This Task?

Visit this book's companion website at resources.corwin.com/minethegap/3-5 for complete, downloadable versions of all tasks.

MINING TIP

We sometimes marvel at students who offer creative, even complicated, jumps on the number line. It's important to remember that we want our students to work towards efficient and accurate strategies.

## WHAT THEY DID

### Student 1

Student 1 shows that he doesn't understand the meaning of the equations. He adds up from one addend to the other. This would be a viable strategy for finding the difference between two numbers on the number line. We can be encouraged that he makes use of friendly numbers in the first prompt.

### Student 2

*Student 2 decomposes the addends into unique chunks.* For the first prompt, he jumps by 50, then 40, and then two jumps of 5. The sequence is equivalent to 100 but slightly more complicated. We can also note that he mixes in two jumps of 5 before then adding a large jump of 300. In the second prompt, he jumps by 200 before breaking apart the remaining 100 to smaller jumps. His mathematics is accurate but inefficient.

**What They Did** *sections analyze how the students' work gives insight into their thinking.*

## USING EVIDENCE

*What would we want to ask these students? What might we do next?*

MINING TIP

We may need to work with physical models and an open number line with two-digit addends before moving to three-digit numbers.

### Student 1

Our first action with Student 1 is to ask him to describe the meaning of the expressions. It is possible that he misread the problem, thinking subtraction instead of addition. Assuming that he did read it correctly, we know that we have work to do to develop understanding of the expressions and the operation. *It would be wise to put the expression into context and work with models of the quantities with base ten blocks* or similar models. We can work to count up by place values, making use of expanded form. We can compare the sum of the base ten blocks with the representation and location on the number line.

MINING HAZARD

We want our students to be efficient mathematicians. The strategies that we develop in them should support their efficiency. We have to be mindful of students who apply strategies both inappropriately and unnecessarily. Student 2 is a good example of the latter.

### Student 2

It is likely that Student 2 is quite comfortable manipulating numbers. His complicated jumping may be a "look what I can do" statement. It's also possible that he has a notion of benchmarks and is trying to navigate them through the computation. For example, on the first number line, he jumps to 408, which is close to the 400 benchmark. His next jump of 40 lands him at mark of 450. *Student 2 serves as a reminder t of our mathematics instruction.* We should c the addend with fewer jumps. We may also w numbers to friendly chunks. For example, 453 or 450 + 3.



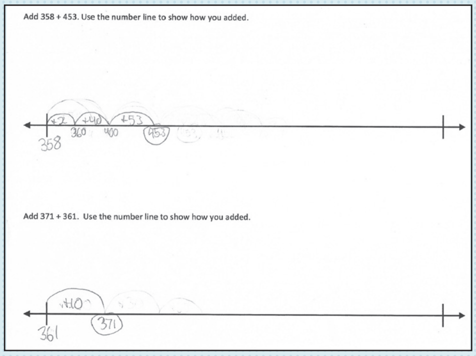
**Mining Tip** icons offer additional notes about mathematics content, misconceptions, or implementing the related tasks.

**Using Evidence** sections identify questions and instructional next steps to address gaps in student understanding.

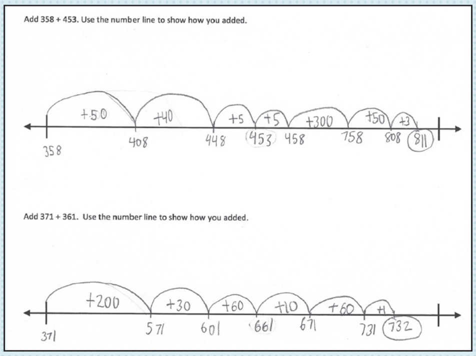
**Mining Hazard** icons also offer insight and advice as to where teachers themselves sometimes go awry in their own thinking.

**TASK 1A:** Use the number lines to add 358 + 453. Use the number line to show how you added. Add 371 + 361. Use the number line to show how you added.

### Student Work 1




### Student Work 2




**Each task is highlighted at the top of the page, with the related student work showcased below.**

## OTHER TASKS

- What will count as evidence of understanding?
- What misconceptions might you find?
- What will you do or how will you respond?

Visit this book's companion website at **resources.corwin.com/minethegap/3-5** for complete, downloadable versions of all tasks.

**TASK 1B: Dennis got out some base ten blocks (235). Jackie got out some base ten blocks (137). How many blocks did Dennis and Jackie get out altogether? Use pictures, numbers, or words to explain your answer.**

Addition within 1,000 should build from conceptual understanding of numbers and the operation as it does with smaller addends. We can use similar tools to develop this understanding. *This problem prompts students to add two three-digit numbers represented by base ten blocks.* The blocks are not arranged in order of place value. Students will need to show that they have made sense of each number. They will also need to show how they found their sum. Some students may simply count all of the blocks for each place value to find the sum. Doing so shows that they understand the meaning of addition, but it also shows that they rely on a lower-level strategy (counting on) and physical models or drawings to combine larger numbers. It will be important to explicitly connect the representations with computation on number lines and equations.

**MODIFYING THE TASK**

Modifying the Task: We can extend the task by asking students to create an addition problem and then represent it with base ten blocks. Students can draw/represent base ten blocks with dots (ones), lines (tens), and squares (hundreds).

**TASK 1C: Kelly added 348 + 256 by breaking both numbers apart. She created 300 + 40 + 8 and 200 + 50 + 6. She said she can then just add the hundreds, tens, and ones to get the sum. Do you agree with Kelly? Will this always work? Create a new equation to show if it will or won't work.**

Flexibly decomposing numbers enhances our ability to compute efficiently and mentally. In this task, students are asked to consider if we can decompose two addends by their place values and then add by their place values. Student understanding of decomposition, in this case expanded form, may be their greatest challenge. Students who understand this will note that you are still adding the same numbers, so it does and always will work. Others will state that there is a difference between the number (348) and the expanded form of it (300 + 40 + 8). The task notes that the numbers are then added by hundreds, then tens, and lastly ones. Students may believe that you can only add by starting with the ones place. Yet, this is only necessary when applying a traditional algorithm. For these students, we can add using expanded form of two addends starting with ones and then add again starting with hundreds, noting that the sum remains the same regardless of which place value we begin with.

**TASK 1D: Use the following hundred chart (701 – 800) to add 732 + 59. Use the following hundred chart (501 – 600) to add 514 + 77.**

Hundred charts are quite useful for adding within 100. We can modify them to model addition within different centuries. In this task, students use a 700 chart and 500 chart to add numbers within the respective century. Some students may count on from one addend by ones. Many students are likely to add by tens and then ones or by ones and then tens. We should look for students who make a jump of a multiple of ten. Doing so shows a more refined approach to computation. We can connect the computation represented on hundred charts with it represented on number lines. This helps students transfer their understanding to new models. We can also connect it to equations to lay the groundwork for developing understanding of symbolic representations and eventually an algorithm.





***Other Tasks* sections provide three additional high-quality tasks related to each Big Idea, along with relevant explanations and analyses.**

***Modifying The Task* marginal notes provide suggestions for further adaptation and exploration.**