MINE THE GAP FOR MATHEMATICAL UNDERSTANDING

THE BOOK AT-A-GLANCE

A quick-reference matrix provides a snapshot of the Big Ideas in the book, along with descriptions of the associated tasks.

BIG IDEAS AND TASKS AT A GLANCE

Big Idea No.	Big Idea	Task	Description
1	Addition and Subtraction of Fractions	1A	Students add mixed numbers on a number line to justify their sums.
1	Addition and Subtraction of Fractions	1B	Students find different fractions with the same sum.
1	Addition and Subtraction of Fractions	1C	Students consider subtraction misconceptions.
1	Addition and Subtraction of Fractions	1D	Students add and subtract fractions to solve a problem.
2	Multiplication and Division of Fractions	2A	Students decompose mixed numbers to multiply.
2	Multiplication and Division of Fractions	2B	Students consider misconceptions about multiplication of fractions.
2	Multiplication and Division of Fractions	2C	Students solve problems with multiplication and division of fractions.
2	Multiplication and Division of Fractions	2D	Students consider the results of dividing with fractions.
3	Reasoning About Addition and Subtraction of Fractions	3A	Students reason about the difference of fractions.
3	Reasoning About Addition and Subtraction of Fractions	3B	Students compare the sums of fractions.
3	Reasoning About Addition and Subtraction of Fractions	3C	Students compare sums of fractions to a benchmark.
3	Reasoning About Addition and Subtraction of Fractions	3D	Students reason about subtraction with mixed numbers.

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CHAPTER 4

RATIO, PROPORTION, AND PERCENT

THIS CHAPTER HIGHLIGHTS HIGH-QUALITY TASKS FOR THE FOLLOWING:

- Big Idea 17: Representing Ratios

  Ratios can be represented in different ways. But ratios can also represent different ideas, including part-to-part and part-to-whole relationships.
- Big Idea 18: Equivalent Ratios

  Equivalent ratios model the same relationship between two quantities. We can use a variety of representations to model this equivalency.
- Big Idea 19: Unit Rates

  Finding and applying unit rates allows us to have a base of comparison between two scenarios. Unit rates also produce a scale factor for finding other equivalent ratios.
- Big Idea 20: Using Ratios to Solve Problems

  Ratio reasoning provides multiple strategies for solving real-world problems.
- Big Idea 21: Reasoning With Percents

  Tasks involving percent build on understanding of ratio reasoning, because percents are values compared to a whole of 100.
- Big Idea 22: Unit Rate as Slope

  Understanding unit rate as slope leads to understanding of linear functions. We can represent this relationship in different ways to determine the constant of proportionality.

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Chapter Overviews highlight and explain the Big Ideas covered in each chapter.

The highlighted task is explained in depth and potential student responses are predicted and described in detail.

Each Big Idea starts by describing one related high-quality task.

Mining Hazard icons signal examples of incomplete thinking that students may encounter.

Pause and Reflect sections invite teachers to think about the task in relation to their practice and their own students.

BIG IDEA 18

BIG IDEA 18
Equivalent Ratios

TASK 18A

Jenny is sure that 10:12 is equivalent to 25:30. Do you agree with Jenny?

Use pictures, numbers, or words to justify your thinking.

About the Task

Ideas about equivalent ratio extend well beyond the ability to calculate equivalency. Representations support students' understanding of the concept of equivalent ratios. *The open-ended nature of this problem allows the student to select his or her own representation and interpretation of each ratio.* The temptation to calculate may be limited due to the nature of the numbers in the ratios. This is especially noteworthy because there may be problems or tasks with numbers that enable students to find convenient "solutions," possibly without deep understanding. We should also note that this task prompts the students to agree or disagree rather than correct flawed ideas. Our students then need to verify their thinking.

MODIFYING THE TASK

We can modify the numbers used in the ratio relative to our students' understanding and number sense development.

MINING HAZARD

Pictorial representations are appropriate ways to show relationships between two quantities, but using them to verify equivalence can be difficult as numbers or relationships become more complex.

Anticipating Student Responses

Students will typically represent the ratios using tape diagrams or double number line models, as these are the clearest way to show equivalence. *Some students may illustrate these ratios with pictorial representations. Some students may rely on contexts to make sense of ratios. Yet in this task, these*

PAUSE AND REFLECT

- **How does this task compare to tasks I've used?**
- **What might my students do in this task?**

Visit this book's companion website at **resources.corwin.com/minethegap/6-8** for complete, downloadable versions of all tasks.

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WHAT THEY DID

Student 1

Student 1's work shows significant misunderstanding. Her work represents two different challenges students might have when working with this problem. *She subtracts $2\frac{1}{2}$ from 10 because she notes that the problem asks "how many more" and she connects this phrase with subtraction.* Her computation is also flawed. She says $\frac{10}{1} - 2\frac{1}{2} = 2\frac{9}{1}$. To make sense of this error, we can presume that she subtracts in either direction to find a result. In other words, she subtracted left to right for the numerator (10 – 1). She subtracts right to left for the denominator (2 – 1). It's likely that she simply brings the whole number over to her "difference."

MINING TIP

Avoid using key words as an instructional approach to problem solving. Doing so can create misconceptions about problems and set students up for incorrect solutions.

Student 2

Student 2 uses the more efficient strategy. She finds the difference of the two days (9 – 5). She then multiplies the difference by $2\frac{1}{2}$. She uses this strategy instead of multiplying both values by $2\frac{1}{2}$ and then finding the difference of the products. Her note about the "key" communicates that each number in the table is multiplied by $2\frac{1}{2}$. She doesn't multiply $4 \times 2\frac{1}{2}$ correctly. Instead, she multiplies 4 × 2 and then adds the half to the product yielding an inaccurate result of $8\frac{1}{2}$.

USING EVIDENCE

What would we want to ask these students? What might we do next?

Student 1

This problem shows that Student 1 needs work with problem solving and computation. We can work on both concepts at the same time. First, we want to develop problem-solving strategies. We can have her work with models and drawings and connect these with equations. Computation work should revisit addition and subtraction of fractions. We would be wise to begin with fractions less than 1 before moving to mixed numbers. It would also be wise to work with less complicated denominators, such as halves, fourths, eighths, and twelfths.

Student 2

Student 2 shows proficiency with solving the problem. She has made sense of the problem and applies an eff
subtracted 9 – 5. Looking
associated days. But after
which is not relevant to the
with fractions and mixed n
include the half in her multi
problem.

MINING HAZARD

We must consider all of a student's work when determining his or her understanding. We can combine students' written thoughts with their diagrams or drawings to establish full understanding. However, additional ideas may not always link their ideas.

What They Did sections analyze how the students' work gives insight into their thinking.

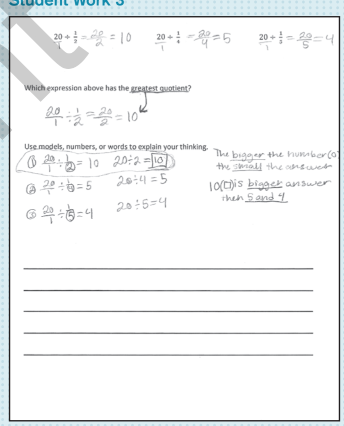
Mining Tip icons offer additional notes about mathematics content, misconceptions, or implementing the related tasks.

Using Evidence sections identify questions and instructional next steps to address gaps in student understanding.

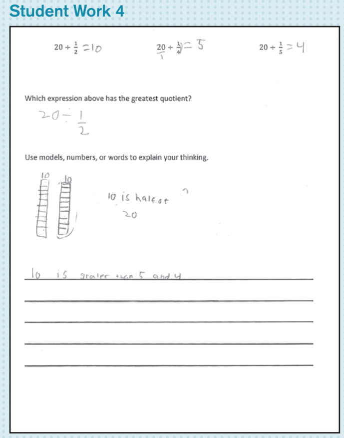
TASK 4A: Which expression has the greatest quotient? Use models, numbers, or words to explain your thinking.

$20 \div \frac{1}{4}$ $20 \div \frac{1}{2}$ $20 \div \frac{1}{5}$

Student Work 3



Student Work 4



Mining Hazard icons also offer insight and advice as to where teachers themselves sometimes go awry in their own thinking.

Each task is highlighted at the top of the page, with the related student work showcased below.

OTHER TASKS

- What will count as evidence of understanding?
- What misconceptions might you find?
- What will you do or how will you respond?

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TASK 4B: Danny knows $50 \times \frac{1}{2} = 25$. Because of this, she knows that $50 \times \frac{1}{4}$ must be less. Do you agree with Danny? What do you think the new product might be? Use pictures, numbers, or words to explain your thinking.

Understanding how the size of factors impact products is critical for determining the reasonableness of our answers. It is likely that some, if not most or even all, of our students believe that multiplication yields a product larger than the first factor, but as we know, this isn't the case. In this task, our students are asked to describe why a product of a factor and $\frac{1}{4}$ will be less than the product of the same factor and $\frac{1}{2}$. Our students are likely to reason that it makes sense because $\frac{1}{4}$ is less than $\frac{1}{2}$. It will be interesting to see if they recognize that the product will be 12.5 (or exactly half of 25). Some students may complete the computation to justify their solution. This may not be indicative of the reasoning we are seeking in our students.

TASK 4C: Oscar noticed these two columns of equations on the board.

$50 \div \frac{1}{10} = 500$	$50 \div \frac{2}{10} = 250$
$40 \div \frac{1}{10} = 400$	$40 \div \frac{2}{10} = 200$
$30 \div \frac{1}{10} = 300$	$30 \div \frac{2}{10} = 150$
$20 \div \frac{1}{10} = 200$	$20 \div \frac{2}{10} = 100$

What patterns do you notice about the equations?

So how can knowing $80 \div \frac{1}{10} = 800$ help you solve $80 \div \frac{2}{10}$?

Intentional arrangement of equations can help our students see patterns in computations. The quotients of the equations are provided so that students can focus on the relationships. We want our students to notice that the quotients of a number and $\frac{2}{10}$ are half of the quotient of the number and $\frac{1}{10}$. Though students may recognize the pattern, they may not be able to explain that they are dividing by twice as much so then the quotient will be half as much. The extension on the task is an opportunity to show that students can generalize their understanding.

MODIFYING THE TASK

This task can be modified to examine patterns when computing other numbers. For example, we could explore the results of multiplying by $\frac{1}{10}$ and $\frac{2}{10}$ by simply changing the operation in the prompt.

Other Tasks **sections provide three additional high-quality tasks related to each Big Idea, along with relevant explanations and analyses.**

Modifying the Task **marginal notes provide suggestions for further adaptation and exploration.**