Introduction

A Brief History of Current State Standards

Contrary to popular belief, academic standards are not new. In fact, they have been around for more than 25 years. The first set of curriculum-specific standards, *The Curriculum and Evaluation Standards for School Mathematics*, was released by the National Council of Teachers of Mathematics in 1989, followed by an updated set of standards, *Principles and Standards for School Mathematics*, in 2001. Both these documents provided a vision for K–12 mathematics by grade-level band. They also formed the foundation for most states' grade-level standards.

In April 2009, the National Governor's Association and the Council of Chief State School Officers met to discuss the creation of the Common Core State Standards Initiative. The purpose of such an initiative was to develop a set of common standards across states to balance the quality of mathematics instruction and learning. Following that meeting, the process of writing the common core standards began. The Standards Development team, led by William McCallum, Phil Daro, and Jason Zimba, included mathematicians, mathematics educators, mathematics education researchers, and classroom teachers. The process included an open invitation for feedback, not only from mathematics educators and associations including the National Council of Teachers of Mathematics but also from the general public. This feedback was considered, and much of it was incorporated into the final document, which was released in June 2010. Following the release of the standards, individual states went through their own processes for reviewing, adopting, and, if necessary, ratifying the adoption of the Common Core State Standards.

Since the release of the Common Core State Standards, most of the states that adopted these standards have made some minor revisions by clarifying a standard, moving a standard to a different grade level, combining standards, and in relatively few cases adding or deleting a standard. A few states have totally revised their standards, and the few states that did not adopt the Common Core State Standards for Mathematics (CCSS-M) continue to work with the standards adopted by their state. Even so, the mathematical content remains similar across states. That is good news so that the relevancy, rigor, and notion of shared standards based on the Common Core have not been lost.

College and Career Ready Standards for Mathematics

The Common Core State Standards (2010) were originally designed to be a clear set of shared goals and expectations for the conceptual knowledge and skills students need in mathematics at each grade level so they can be prepared to succeed in college, career, and life. Keep in mind that most states have maintained the core standards from the original document, while at the same time some states have clarified, combined, or moved some of the standards to a different grade level. In fewer cases, standards have been added or deleted. In this book, we have designed a tool for teachers—the index in the front of this book—to find their state standards, see how they relate to the Common Core standard, and then find the relevant content in this book that will help them better understand what instruction around any mathematical content might look and sound like in the classroom.

The CCSS-M, updated state standards, revised state standards, and rewritten state standards all have a few things in common. They included two critical components of learning mathematics: the content standards and the Standards for Mathematical Practice, sometimes referred to as process standards.

The first group of standards—the content standards—explicitly outline the mathematics we want students to know and be able to do at each grade level. In all cases, the content standards are fewer in number than most previous state standards, and the expectation is that students will develop deeper understanding of that content so less time is spent on re-teaching from year to year. Additionally the standards have been constructed to show connections among ideas at a grade level as well as vertical progressions across grades. For example, you will find that the standards in Grade 6 develop from the mathematical work that students have completed in previous grades. Similarly, the standards in Grade 8 develop from work completed in Grades K–7. Thus it is important for teachers to be knowledgeable of the standards not only at the level they are teaching but also at the preceding grade level and the next grade level. This continuity is present across all states' standards.

The second group—the Standards for Mathematical Practice, referred to as process standards in some states—describes the habits of mind that students should develop as they do mathematics. Some states have maintained the National Council of Teachers of Mathematics' (NCTM) original nomenclature of process standards, but the underlying principles are the same. These standards remain the same across all grade levels, K–12. As teachers plan mathematics lessons they should consider how students will use the practices in learning and doing mathematics.

True across all states is that the standards *are not* intended to serve as a curriculum. Decisions about mathematics programs, textbooks and materials, sequencing topics and units, and instructional frameworks are left for local and state school districts to make. They do not tell teachers how to teach. It is important to remember that they describe what students need to know and be able to do. Schools and teachers know best how to help students reach both the content and the practice standards.

Further, there is no universal body that dictates specific assessments. Some states use assessments developed by SBAC (Smarter Balanced Assessment Consortium) or PARRC (Partnership for Assessment of Readiness for College and Careers). Others have developed and are using their own assessments. Other facts and information can be found at http://www.corestandards.org or on your state department of education website.

Instructional Shifts

While the standards do not call for a particular instructional model or philosophy, they are based on the best of existing standards. What is different is that they call for specific instructional shifts: focus, coherence, and rigor.

The content standards call for greater *focus* on fewer topics. An examination of the mathematics standards of high-performing countries indicate that fewer, more focused topics at a grade level allow students to deepen their understanding of the mathematics and gain a stronger foundation for ongoing study of mathematics. Within the standards, the major mathematical work of each grade level has been identified (www.corestandards.org). That means that not all the content within a grade is emphasized equally among the content standards. The list of content standards for a grade is neither linear nor is it a checklist. Some clusters require greater emphasis than others. They take more time for students to master with depth of understanding. At least 65% to 85% of instructional time should focus on the major work for each grade level. Areas of major work include the following:

In Grade 6: Instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) understanding division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

In Grade 7: Instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

In Grade 8: Instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) understanding the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance.

This does not mean that other standards should be skipped. Rather, the supporting standards should be taught to connect mathematical ideas to the essential standards. The additional standards provide students with experiences that will be foundational to work in future grades. Neglecting material will leave gaps in student skill and understanding.

Many of us learned mathematics as a set of disconnected topics, with much of our skill based on tricks ("Ours is not to reason why, just invert and multiply") or mnemonic devices (Please Excuse My Dear Aunt Sally). In reality, mathematics is a coherent body of knowledge made up of topics that are connected and build on each other. The call for *coherence* in the content standards ensures that there are carefully constructed progressions from grade to grade so students build new understandings on the foundations built in previous years. Each standard is not a new topic, but an extension of previous learning. In addition to the progressions across grade levels, the standards incorporate specific connections within a grade level. For example, as students develop conceptual understanding of multiplication and division, the relationship of these operations to each other is consistently reinforced through building conceptual understanding, procedural skills, and applying these understanding and skills to various contexts.

The final instructional shift, *rigor*, refers to how we support students in developing deep understanding of each standard. Understanding does not develop by assigning more worksheets or more difficult examples and problems. Rather, it calls for instructional practice that balances conceptual understanding, procedural skills, and applying mathematical ideas to a variety of contexts.

The following descriptions of each component of rigor come from the standards document, which can be found at www.corestandards.org.

> *Conceptual understanding:* The standards call for conceptual understanding of key concepts, such as place value. Students must be able to access concepts from a number of perspectives to see mathematics as more than a set of rules or procedures.
>
> *Procedural skills and fluency:* The standards call for speed and accuracy in calculation. Students must practice core skills, such as basic facts and addition/subtraction computation, to have access to more complex concepts and procedures. Fluency is built on conceptual understanding and, with young children, through the development of ideas through representations using concrete materials, pictures, numbers, and words.

Application: The standards call for students to use mathematics in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

Terminology Used in This Book

The language of the Common Core and other standards differs from traditional standards. Familiarity with the structure, descriptive terminology, and functions of each section will help you make the best use of your standards. While what follows describes the Common Core Standards' structure, most states have utilized the same description in their updated standards. You will need to check your individual state standards to find any changes to these descriptors. Numbering and identification systems for your state standards may also be slightly different; the index in the front of this book will help you correlate your standards with the original numbering system of the Common Core Standards.

Standards define what students should understand and be able to do.

Clusters summarize groups of related standards. Be aware that standards from different clusters may sometimes be closely related as mathematics is a connected subject.

Domains are larger groups of related standards. Be aware that standards from different domains may sometimes be closely related.

Domain

Ratios and Proportional Relationships 6.RP

Understand ratio concepts and use ratio reasoning to solve problems.

Cluster

Standard

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

[1] Expectations for unit rates in this grade are limited to non-complex fractions.

Source: Common Core State Standards for Mathematics (www.corestandards.org).

As districts develop units of study for a grade level, careful consideration should be given to the order and connection among topics and standards.

The Standards for Mathematical Practice (Processes) describe the habits of mind teachers must incorporate into classroom instruction to develop depth of understanding of critical mathematical concepts in their students. The mathematical practices are not intended to be taught in isolation but should be integrated into daily lessons. Some lessons may focus on developing one or two of these standards; others may incorporate seven or all eight standards. Note that you do not "teach" these standards. Rather, they are the type of mathematical thinking and doing that we want students to practice as they are developing mathematical understanding.

Throughout the following chapters, we have included examples of mathematical practice that can be used in each cluster. These are not meant to limit lessons to using only the practices. They are examples of key practices that should be included in lessons around that particular cluster. It is likely that you will use all the practices throughout the cluster and domain.

These practices, briefly explained on the next page, are essential for student success. If students are actively engaged in using the eight practices, they are learning rigorous, meaningful mathematics. Keep in mind that your state may have combined some of these or reverted to the five processes from NCTM's Principles and Standards for School Mathematics (2000). Whether you are dealing with the practice standards or the process standards, you will find the foundational practices (problem solving, communication, connections, representations, and reasoning and proof) to be very similar across all these standards.

SFMP 1. Make sense of problems and persevere in solving them.

Students work to understand the information given in a problem and the question that is asked. They choose a strategy they can use to find a solution and check to make sure their answer makes sense.

SFMP 2. Reason abstractly and quantitatively.

Students make sense of quantities and their relationships in problem situations. At this level, students can model problem solutions.

SFMP 3. Construct viable arguments and critique the reasoning of others.

Students in Grades 6–8 often use stated assumptions, definitions, and established results in constructing arguments. At this level, students must justify and communicate their conclusions as well as listen to other students' explanations.

SFMP 4. Model with mathematics.

Students use various representations, models, and symbols to connect conceptual understanding to skills and applications. Students should use the mathematics they know to solve problems in everyday life, society, and the workplace. At this level, students may write an equation or may connect representations and explain the connections.

SFMP 5. Use appropriate tools strategically.

Students consider the available tools when solving a mathematics problem. At times, students may choose to solve problems with mental calculations, with paper and pencil, or with other technology.

SFMP 6. Attend to precision.

Students communicate precisely with others. Students in Grades 6–8 explain their knowledge of mathematical symbols that explicitly connect to using the correct mathematical vocabulary.

SFMP 7. Look for and make use of structure.

Students look closely to discern patterns and structure.

SFMP 8. Look for and express regularity in repeated reasoning.

Students notice if calculations are repeated and make generalizations. At this level, students discover shortcuts through making the generalizations and understanding why they work.

Effective Teaching Practices

Quality mathematics teaching is a critical key for student success. In *Principles to Actions* (2014), the National Council of Teachers of Mathematics outlines eight valuable teaching practices every teacher should incorporate to guarantee student achievement. These eight research-informed practices, briefly explained below, provide a foundation for effective Common Core Mathematics teaching and student learning.

1. Establish mathematics goals to focus learning.

Establishing learning goals sets the stage and helps guide instructional decisions. Teachers must keep in mind what is to be learned, why the goal is important, and where students need to go (the trajectory), as well as how learning can be extended. Students must clearly understand the purpose of each lesson beyond simply repeating the standard.

2. Implement tasks that promote reasoning and problem solving.

Implementing tasks that promote reasoning and problem solving provides opportunities for students to engage in exploration and encourages students to use procedures in ways that are connected to conceptual understanding. The tasks teachers choose should be built on current student understandings and have various entry points with multiple ways for the problems to be solved.

3. Use and connect mathematical representations.

Using and connecting representations leads students to deeper understanding. Different representations including concrete models, pictures, words, and numbers should be introduced, discussed, and connected to support students in explaining their thinking and reasoning.

4. Facilitate meaningful mathematical discourse.

Facilitating meaningful student mathematical conversations provides students with opportunities to share ideas, clarify their understanding, and develop convincing arguments. Talking and sharing aloud can advance the mathematical thinking of the whole class.

5. Pose purposeful questions.

Posing purposeful questions reveals students' current understanding of a concept and encourages students to explain, elaborate, and clarify thinking. Asking good questions makes the learning of mathematics more visible and accessible for student examination.

6. Build procedural fluency from conceptual understanding.

Building procedural fluency from conceptual understanding based on experiences with concrete representations allows students to flexibly choose from a variety of methods to solve problems.

7. Support productive struggle in learning mathematics.

Supporting productive struggle in learning mathematics is significant and essential to learning mathematics with understanding. Productive struggle allows students to grapple with ideas and relationships. Giving young students ample time to work with and make sense out of new ideas is critical to their learning with understanding.

8. Elicit and use evidence of student thinking.

Eliciting and using evidence of student thinking helps teachers access learning progress and can be used to make instructional decisions during the lessons as well as help prepare what will occur in the next lesson. Formative assessment through student written and oral ideas are excellent artifacts to assess student thinking and understanding.

How to Use This Book

The purpose of this book is to help teachers more deeply understand the mathematical meaning of each cluster and standard within the domains of Grades 6–8. We want this book to be your toolkit for teaching the mathematics standards and we have left ample space for you to take notes and add ideas and other resources you have found to be helpful.

You will find that each section is made up of one Domain and begins with an overview of how the domain progresses across sixth, seventh, and eighth grade. A list of helpful materials, black line masters (BLM), and key vocabulary from the domain is included in the overview as well.

We track each domain across sixth, seventh, and eighth grade with a page for each cluster and the standards within that cluster. A description of the cluster and how the Standards for Mathematical Practice can be incorporated into your teaching of the cluster concepts follows. Since the standards are intentionally designed to connect within and across domains and grade levels, a list of related standards is included in the cluster overview. We suggest that as you prepare work on a cluster you look at these standards to have a better idea of the mathematics students learned in previous grades and where they are going in future grades. A list of all the standards is found inside the front and back covers.

Each standard within a cluster is explained with an example of what the teacher does to work with that standard in the classroom followed by what the students do. It is important to note that most standards will take several days, and it is likely that you will be connecting across standards and domains as you teach for understanding.

Addressing student misconceptions and common errors in developing student understanding of a concept concludes the contents for each standard.

Each cluster ends with a template for what to consider when planning instruction for that cluster. A BLM of the template guide is included in the resource section for duplication and use with additional standards. This is also downloadable from **resources.corwin.com/yourmathcompanion6-8**. A sample planning page for each domain at each grade level has been included as the last page in each grade level domain.

In the Resources section, you will find Table 1, on the Standards for Mathematical Practice, and Table 2, which covers the Effective Teaching Practices. You will also find BLM for key materials that you can photocopy or download from **resources.corwin.com/yourmathcompanion6-8**. These are designed to be samples and we encourage you to use them or redesign them to best meet the needs of your students. A list of our favorite resource books and high-quality online resources that are particularly useful to developing mathematical ideas in Grades 6–8 are also included in the Resources section.

Most important, we have aligned all these materials with the updated standards from most states not identified as a Common Core State. If you teach in a state that is not using the Common Core Standards, you can find the standards listed for your state and your grade in the beginning of this book. Read across the table to see how any given standard relates to the original Common Core version and to see where to find the guidance in this book to help you more deeply understand how a mathematical concept or skill should be developed with your students. You'll note that in some cases there are uncorrelated or differently correlated standards, and in some cases the standards have been moved up or down a grade and the relevant information may be found in *Your Mathematics Standards Companion* for Grades 3–5.

We believe that this can become your mathematics standards bible! Read it and mark it with questions, comments, and ideas. We hope that it will help you use these standards and good teaching practice to lay the essential foundation that will ensure your students success in your grade and in all their future study of mathematics.

Reflection Questions

1. What instructional shifts have you seen in your own state standards, and how do they differ from your current instructional practice?

2. The Standards for Mathematical Practice describe the habits of mind that students need for thinking about and doing mathematics. While not every practice will be in every lesson, select one standard at your grade level and consider some ways you can incorporate these practices in a lesson for that standard.
   - How will these practices provide you with information about student understanding?
   - How will this help you to better assess students?
   - How will this information help you in planning lessons?

3. The Effective Teaching Practices describe specific actions that teachers must consider in planning and implementing lessons and assessing student performance.
   - How are the practices connected? Work with colleagues to plan a lesson that employs all of these practices.
   - How can you modify a traditional task so that it promotes reasoning and problem solving?
   - What representations will help students more deeply understand the concept?
   - How will you connect conceptual understanding to build procedural fluency?
   - What kinds of information will you look for to help inform your instruction?

   (For more information on the Effective Teaching Practices, go to www.nctm.org.)